A Parallelization Algorithm for Adequacy Assessment of the Electrical Grid

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European Commission

Joint Research Centre

Table of Contents

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Where $\mathcal{V}(x,\omega)$ is the solution to (ED) in function of the expanded capacities x and the scenario ω.

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Economic Dispatch (ED) model Scary Slide

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$$L^{\min}_{n,l} \leq f_{n,l,t,\omega} \leq L^{\max}_{n,l}$$

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This can be solved efficiently with L-shaped or subgradient schemes.

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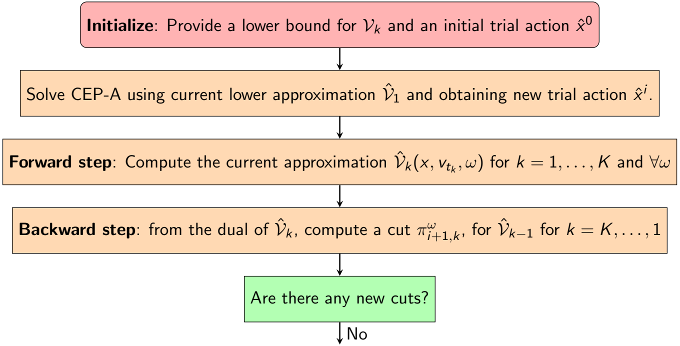

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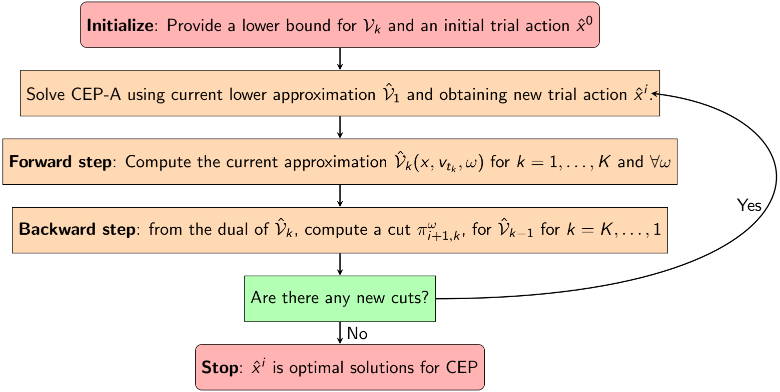
Are there any new cuts?

↓ No

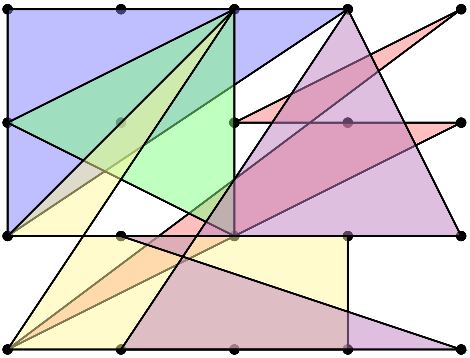
Stop: $\hat{x}^i$ is optimal solutions for CEP

Literature review - 3/3 Algorithm description



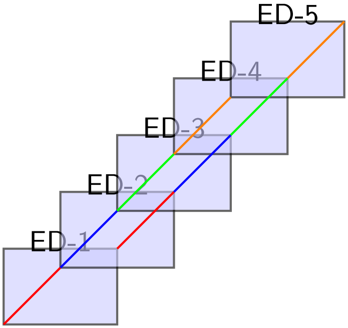
Definition: The *hypergraph* associated to a linear programming problem LP, denoted by $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, is constructed as follows:

- The *nodes* $\mathcal{N}$ of $\mathcal{G}$ correspond to the variables of the LP.
- The *hyperedges* $\mathcal{E}$ of $\mathcal{G}$ correspond to each set of variables that appears together in any constraint of the LP.



Example of LP hypergraph.

Model relaxation description: Intermediate Economic Dispatches (ED-k)



(ED) hypergraph representation.

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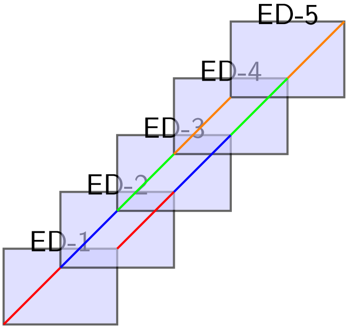

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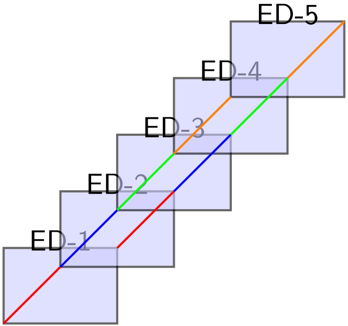

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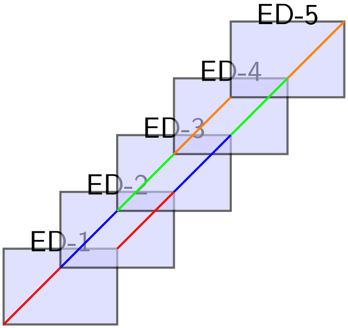

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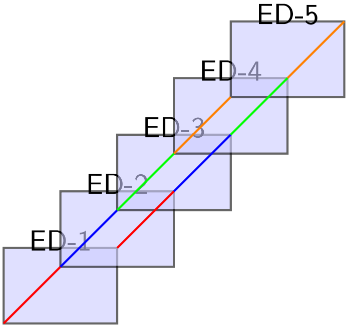

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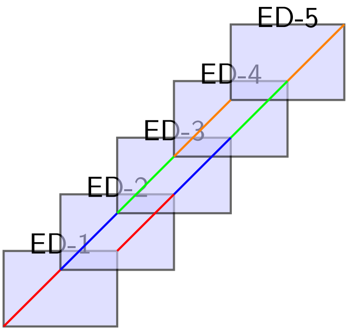
Model relaxation description: Intermediate Economic Dispatches (ED-k)



(ED) hypergraph representation.

- We divide the time horizon into K intervals: $\{t_0 := 0, \dots, t_1\}$, $\{t_1 + 1, \dots, t_2\}$, $\dots$ $\{t_{K-1} + 1, \dots, t_K = T\}$
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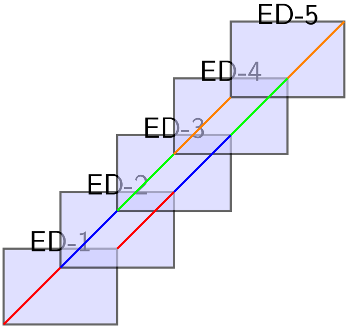

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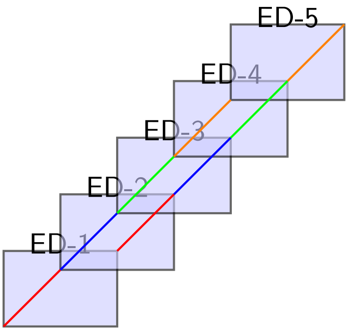

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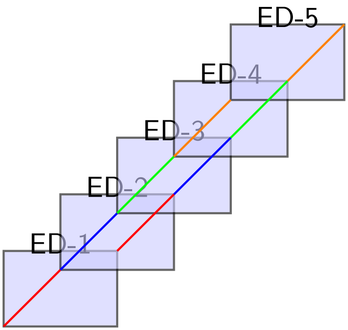

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- The corresponding optimal values are $\mathcal{V}_{\mathbf{k}}(\mathbf{x}, \mathbf{v}_{\mathbf{t_k}}, \mathbf{v}_{\mathbf{t_{k+1}},\omega})$

Observation

$$\mathcal{V}(x,\omega) = \min_{\{v_{t_k}\}_{k=1}^{K}} \sum_{k=0}^{K-1} \mathcal{V}_k(x, v_{t_k}, v_{t_k+1}, \omega) \tag{5}$$

Model relaxation description: Lower Approximation of (ED)

Since each function $\mathcal{V}_k$ is piecewise linear convex in $x, v_{t_K}, v_{t_{K+1}}$, it can be approximated by a collection of supporting hyperplanes $\{\pi^w_{i,k}(x, v_{t_k}, v_{t_{k+1}})\}$ of each $\mathcal{V}_k$.

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An approximation of (ED) is given by:

$$\begin{aligned}
\hat{\mathcal{V}}(x,\omega) &= \min_{\{v_{t_k}\}_{k=1}^K} \sum_{k=0}^{K} \hat{\mathcal{V}}_k(x, v_{t_k}, v_{t_{k+1}}) = \\
&= \min_{\{v_{t_k}\}_{k=1}^K} \sum_{k=0}^{K} \theta_k^\omega \\
&\quad \text{s.t.} \quad \theta_k^\omega \geq \pi^\omega_{i,k}(x, v_{t_k}, v_{t_{k+1}}) \quad \forall i, k
\end{aligned} \tag{ISP}$$

We refer to this problem as the **Intermediate Storage Problem (ISP)**

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$$\begin{aligned}\hat{\mathcal{V}}(x,\omega) = \min_{\{v_{t_k}\}_{k=1}^K} \sum_{k=0}^K \hat{\mathcal{V}}_k(x, v_{t_k}, v_{t_{k+1}}) = \\ = \min_{\{v_{t_k}\}_{k=1}^K} \sum_{k=0}^K \theta_k^\omega \\ \text{s.t.} \quad \theta_k^\omega \geq \pi^\omega_{i,k}(x, v_{t_k}, v_{t_{k+1}}) \quad \forall i, k\end{aligned} \tag{ISP}$$

We refer to this problem as the **Intermediate Storage Problem (ISP)**
(I know, very original)

Model description: Relaxed Capacity Expansion(CEP-R)

$$\min_{x} c'x + \mathbb{E}_{\omega}\left[\mathcal{V}(x,\omega)\right]$$
$$\text{s.t. } 0 \leq x_{n,g} \leq X_{n,g} \tag{CEP}$$

Model description: Relaxed Capacity Expansion(CEP-R)

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\end{aligned} \tag{CEP}
$$

$$
\begin{aligned}
&\min_x \; c'x + \mathbb{E}_w \left[\hat{\mathcal{V}}(x,w)\right] \\
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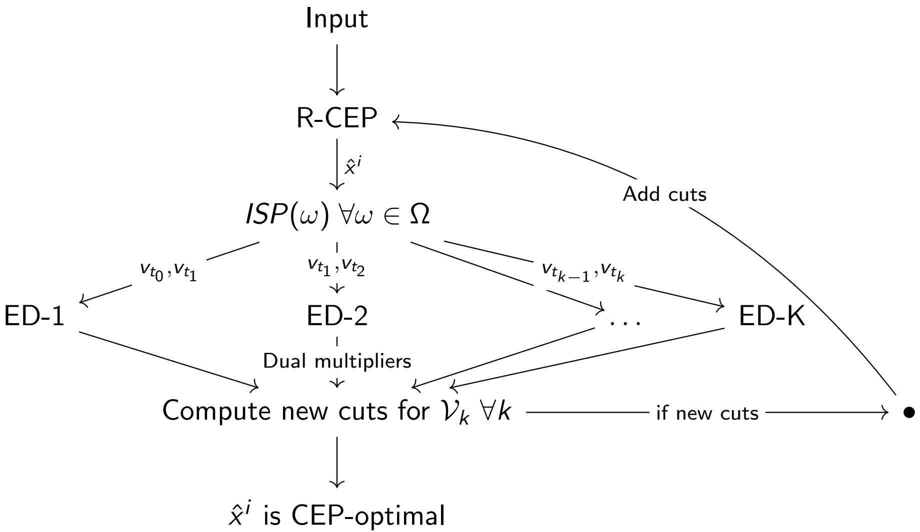
Since calculating $\hat{\mathcal{V}}$ is straightforward, solving (CEP-R) can be done efficiently with L-shaped or subgradient schemes.

Algorithm


Input
R-CEP
$\hat{x}^i$
$ISP(\omega)\ \forall\omega \in \Omega$
Add cuts
v_{t_0}, v_{t_1}
v_{t_1}, v_{t_2}
$v_{t_{k-1}}, v_{t_k}$
ED-1
ED-2
. . .
ED-K
Dual multipliers
Compute new cuts for $\mathcal{V}_k\ \forall k$
if new cuts
$\hat{x}^i$ is CEP-optimal

Convergence results - 1/5

- Since $(CEP - R) \leq (CEP)$ if a $(CEP - R)$ optimal solution is (CEP) feasible then it's also (CEP)-optimal.

Convergence results - 1/5

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Remark 1: It is sufficient to prove that after a finite number of steps (i) of the algorithm we have:

$$\hat{\mathcal{V}}(\hat{x}^i, \omega) = \mathcal{V}(\hat{x}^i, \omega) \text{ for all } \omega \in \Omega \tag{6}$$

Convergence results - 2/5

Observation

After a finite number of iterations no new cuts are found for $\mathcal{V}_k$*.*

Proof.

$$(7)$$

Convergence results - 2/5

Observation

After a finite number of iterations no new cuts are found for $\mathcal{V}_k$.

Proof.

$$\#\{p \mid p \text{ is a normal vector of a supporting hyperplane of } \mathcal{V}_k\} \leq \tag{7}$$

Convergence results - 2/5

Observation

After a finite number of iterations no new cuts are found for $\mathcal{V}_k$.

Proof.

$$\#\{p \mid p \text{ is a normal vector of a supporting hyperplane of } \mathcal{V}_k\} \leq$$
$$\#\{\text{dual solutions } p = q'B^{-1} \text{ of (ED-k) for varying } x, v_{t_k}, v_{t_{k+1}}\} \leq \tag{7}$$

Convergence results - 2/5

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$$
\begin{aligned}
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&\#\{\text{basis matrices of (ED-k)}\} < \infty
\end{aligned}
\tag{7}
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Convergence results - 2/5

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Convergence results - 2/5

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After a finite number of steps:

- new cut: $\bar{c}(x, v) = p'(x, v) + b$

Convergence results - 2/5

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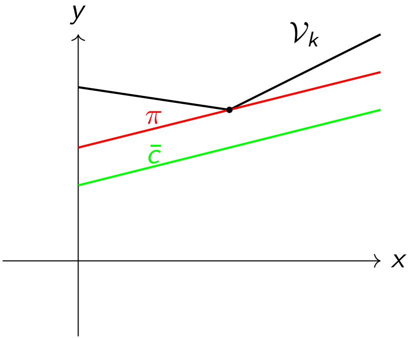
After a finite number of steps:

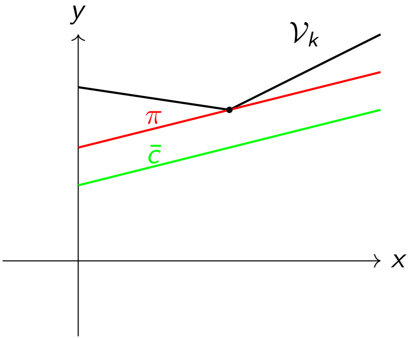
- new cut: $\bar{c}(x, v) = p'(x, v) + b$
- an old cut: $\pi(x, v) = p'(x, v) + \bar{b}$

□

Convergence results - 3/5



Convergence results - 3/5



Since both are supporting hyperplanes it follows that $b = \bar{b}$ (and therefore $\bar{c}$ is not a new cut).

Convergence results - 4/5

Observation

If after the i-iteration no new cuts are added for some i and k then $\hat{\mathcal{V}}_k(\hat{x}^i, \hat{v}_k, \hat{v}_{k+1}) = \mathcal{V}_k(\hat{x}^i, \hat{v}_k, \hat{v}_{k+1})$.

Convergence results - 4/5

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Proof.

Let $\bar{c}_k^{\omega}(x, v_{t_k}) := p'(x - \hat{x}^i, v_{t_k} - \hat{v}_{t_k}) + \mathcal{V}_k(\hat{x}^i, \hat{v}_{t_k})$ be the new cut found after the i-th iteration.

Convergence results - 4/5

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Convergence results - 4/5

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$$\mathcal{V}_k(\hat{x}^i, \hat{v}_{t_k}) \geq \hat{\mathcal{V}}_k(\hat{x}^i, \hat{v}_{t_k}) \geq \bar{c}(\hat{x}^i, \hat{v}_{t_k}) = \mathcal{V}_k(\hat{x}^i, \hat{v}_{t_k})$$

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which concludes the proof. □

Convergence results - 5/5

In conclusion, we have $\hat{\mathcal{V}}_k(\hat{x}^i, v_{t_k}, v_{t_{k+1}}, \omega) = \mathcal{V}_k(\hat{x}^i, v_{t_k}, v_{t_{k+1}}, \omega)$ for all ω, k.

Convergence results - 5/5

In conclusion, we have $\hat{\mathcal{V}}_k(\hat{x}^i, v_{t_k}, v_{t_{k+1}}, \omega) = \mathcal{V}_k(\hat{x}^i, v_{t_k}, v_{t_{k+1}}, \omega)$ for all ω, k.
Thus $\hat{\mathcal{V}}(\hat{x}^i, \omega) = \mathcal{V}(\hat{x}^i, \omega)$.

Convergence results - 5/5

In conclusion, we have $\hat{\mathcal{V}}_k(\hat{x}^i, v_{t_k}, v_{t_{k+1}}, \omega) = \mathcal{V}_k(\hat{x}^i, v_{t_k}, v_{t_{k+1}}, \omega)$ for all ω, k.
Thus $\hat{\mathcal{V}}(\hat{x}^i, \omega) = \mathcal{V}(\hat{x}^i, \omega)$.

Proposition

The algorithm converges after a finite number of iterations and $\hat{x}^i$ is an optimal solution for (CEP).

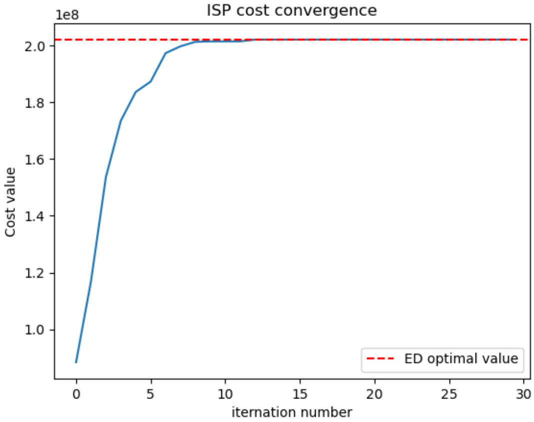
Initial results - 1/2

We implemented the algorithm on the following network, consisting different kinds of storage units, solar, gas and wind power for a time horizon of 5 weeks and time steps of one hour.

Network layout

Initial results - 2/2

In this instance the (not parallelized) algorithm converges to the optimal solutions in 12 iterations and in 0.46 seconds. Benders' algorithm converged in 0.44 seconds.



Objective value of (ISP) for each iteration.

Conclusions.

- The specific structure of the intertemporal constraints makes it possible do develop tailored optimization algorithms for (CEP).

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- The algorithm is analogous to a three stage bender decomposition. (And I think the work by Filippo Pecci presented yesterday.)

Future Work.

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- In general: equivalent LP formulations give different corresponding Hypergraph with different degrees of parallelizability.

Thank you for your attention.

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https://www.compopt.it

Some references:

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Power Grid Optimization

Stochasticity ↓

Time / Exactness ↑

- Optimal Power Flow (OPF) [Bie+20]
  - AC OPF: exact physical model
  - Security-Constrained OPF (SCOPF) – Includes contingencies to guarantee system security under failures.
  - DC OPF and other linearized models [BM14]
  - other relaxations.
- Unit Commitment – Determines on/off status of power units, ignoring grid constraints.
- Economic Dispatch (ED) – Minimizes generation cost, ignoring grid constraints.

Capacity expansion problem: Based on Economic Dispatch models with added flow balance at bus nodes and various scenarios.