POLITECNICO MILANO 1863

DIPARTIMENTO DI ENERGIA

Università di Pavia

Transient Stability Constrained Optimal Power Flow Problems

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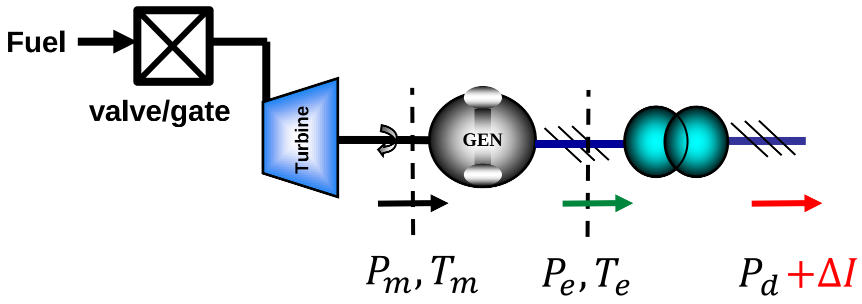
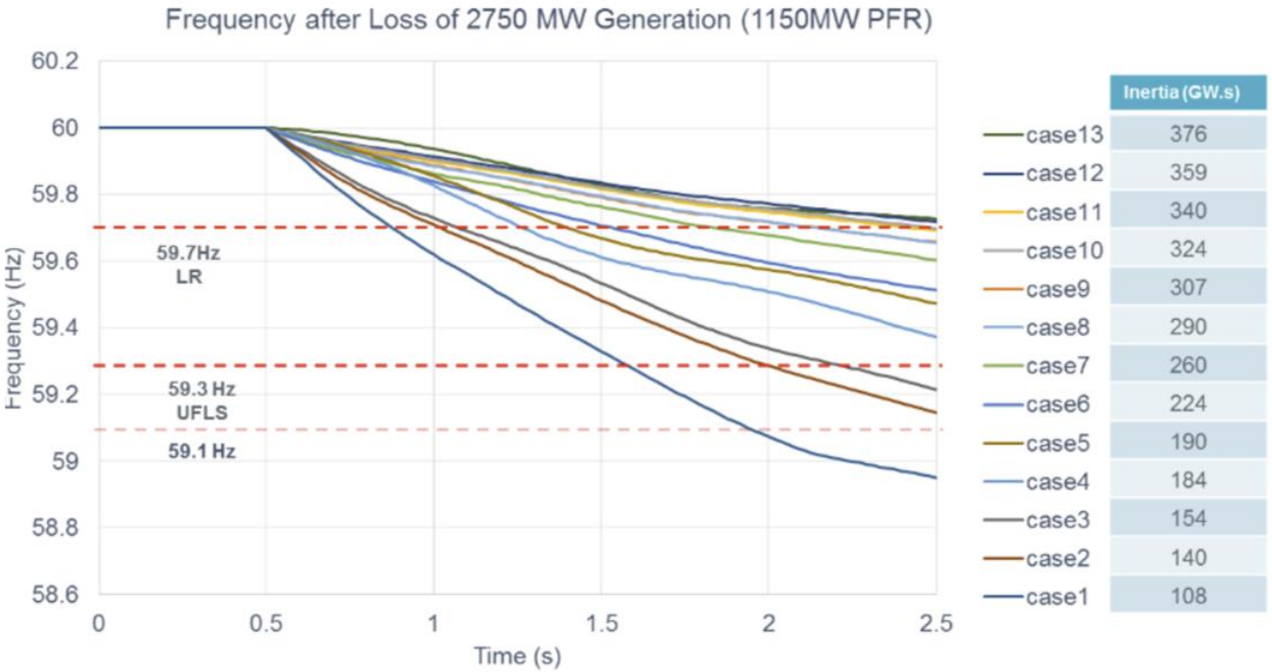
Motivation

- Standard Security Constrained – OPF (SCOPF) problems give an optimal solution that guarantees steady-state security (control and state variables in-bounds, N-1 security criteria etc.)

- the increase of *converter-based* clean generation resources substitutes the *rotating-machine* based generation lowering the available system inertia and negatively affecting system's dynamic security:



$$T_m - T_e = J\frac{d\omega}{dt} = 0$$



- It is thus necessary to formulate an SCOPF that also considers Transient Stability aspects, in other words, a Transient Stability Constrained OPF (TSC-OPF)

$\min C(p)$ → p is the decision variable vector $p = (P_g, Q_g, V, \theta)^T$

such that

$g_s(p) = 0$ → set of the **steady-state** equality constraints (PF eq, branch currents, etc)

$h_s(p) \geq 0$ → set of the **steady-state** inequality constraints (capability, box bounds, N-1 security conditions etc)

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$\min C(p)$ → p is the decision variable vector $p = (P_g, Q_g, V, \theta)^T$

such that

$\dot{x} = f(x(t), y(t), p)$ → x is the vector of state variables (rotor angles, generators frequencies, etc.)
y is the vector of algebraic link variables
dot operator is the derivative wrt. time

$0 = g(x(t), y(t), p)$ → link algebraic equations

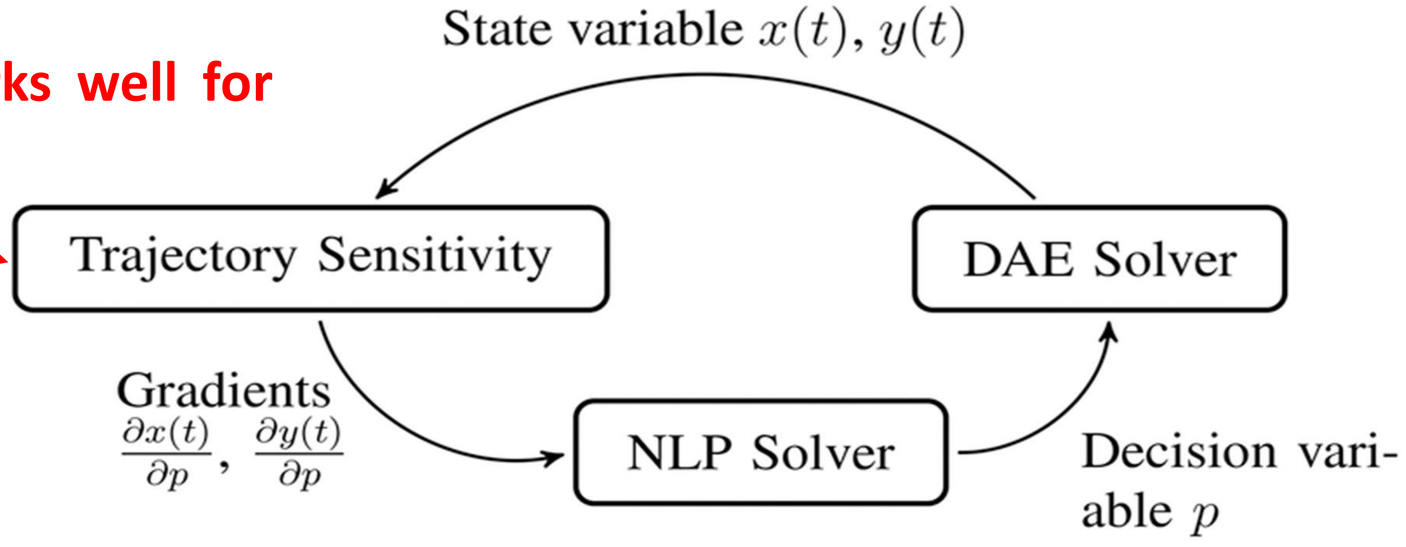
$x(t_0) = I_{x0}$ → set of initial conditions for x

$y(t_0) = I_{y0}$ → set of initial conditions for y

$\boldsymbol{h(x(t), y(t)) \leq 0}$ → **stability constraints**

- **Constraint transcription** is an algorithmic framework that decouples optimization algorithms and simulation tools [*]. Differential equations are integrated outside the optimization process and interfaced with NLP solvers:

!!! Anything but trivial (works well for small systems, though)



[*] S. Abhyankar, G. Geng, M. Anitescu, X. Wang, and V. Dinavahi, "Solution techniques for transient stability-constrained optimal power flow - Part I," IET Gener. Transm. Distrib., vol. 11, no. 12, pp. 3177–3185, 2017

- *Integration into optimization problem:*

$$H(x(p,t), y(p,t)) = \sigma \int_0^{T} [\max(0, h(x(p,t), y(p,t))]^{n} \mathrm{d}t = 0$$

- *Remarks:*

  - $x(p,t)$ and $x(p,t)$ are expressed, using trajectory analysis, as an approximation valid in the vicinity of p_0, the previous NLP optimal solution
  - remember that $h(x(t), y(t))$ is the function quantifying the stability conditions
  - **!!!** σ, μ are user defined parameters to assure smoothness of convergence
  - the equality condition is, in general, to hard and may lead to divergence of NLP

- *Integration into optimization problem - relaxation*

$$H(x(p,t) - \rho, y(p,t) - \rho) \leq 0$$

  - where ρ is a vector of positive slack variables minimized in the objective function

- *Simulation discretization:* the differential equations for all time steps are discretized to non-linear algebraic equations by using a numerical integration scheme.

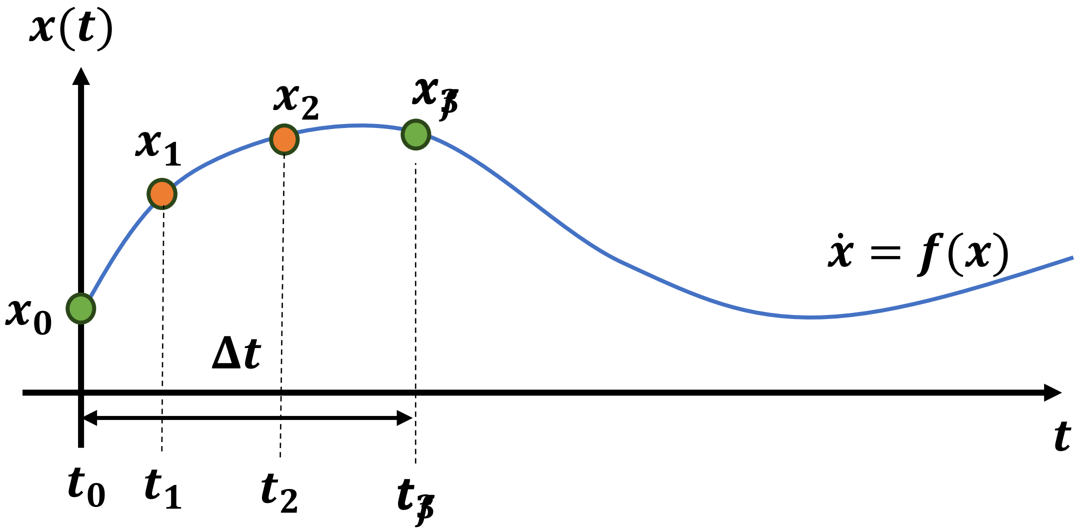
  - in *power systems* optimization the few work available have used the Taylor integration scheme for this

$$x(t) - x(t - \Delta t) = \frac{\Delta t}{2} \cdot [f(x(t), y(t), p) - f(x(t - p), y(t - p), p)]$$

where Δt is the integration step

- The solution of the differential equations at discrete time points is approximated by a Lagrange interpolating polynomial

$$x(t) = A + Bt + Ct^2 + Dt^3$$



[**] J. D. Hedengren, R.A. Shishavan, K. M. Powell, T. F. Edgar, "Nonlinear modeling, estimation and predictive control in APMonitor," Computers & Chemical Engineering, vol. 70, 2014, pp. 133-148.

- The objective is to determine a matrix M that maps the derivatives to the non-derivative values:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = M \left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} - \begin{bmatrix} x_0 \\ x_0 \\ x_0 \end{bmatrix} \right)$$

- Time points for each interval are chosen according to *Lobatto quadrature*; In the case of 4 nodes per horizon step, the internal values are chosen at $t_{1,2} = \frac{1}{2} \pm \frac{\sqrt{5}}{10}$; time points are shifted to a reference time of zero (t_0 = 0) and a final time of t_f = 1

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- Substituting the polynomial into the mapping relationship:

$$\begin{bmatrix} B + 2Ct_1 + 3Dt_1^2 \\ B + 2Ct_2 + 3Dt_2^2 \\ B + 2Ct_3 + 3Dt_3^2 \end{bmatrix} = M \begin{bmatrix} Bt + Ct_1^2 + Dt_1^3 \\ Bt + Ct_2^2 + Dt_2^3 \\ Bt + Ct_3^2 + Dt_3^3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2t_1 & 3t_1^2 \\ 1 & 2t_2 & 3t_2^2 \\ 1 & 2t_3 & 3t_3^2 \end{bmatrix} \begin{bmatrix} B \\ C \\ D \end{bmatrix} = M \begin{bmatrix} t_1 & t_1^2 & t_1^3 \\ t_2 & t_2^2 & t_2^3 \\ t_3 & t_3^2 & t_3^3 \end{bmatrix} \begin{bmatrix} B \\ C \\ D \end{bmatrix} \Rightarrow M = \begin{bmatrix} 1 & 2t_1 & 3t_1^2 \\ 1 & 2t_2 & 3t_2^2 \\ 1 & 2t_3 & 3t_3^2 \end{bmatrix} \begin{bmatrix} t_1 & t_1^2 & t_1^3 \\ t_2 & t_2^2 & t_2^3 \\ t_3 & t_3^2 & t_3^3 \end{bmatrix}^{-1}$$

- Now, the mapping relationship is fully determined:

$$M^{-1} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} - \begin{bmatrix} x_0 \\ x_0 \\ x_0 \end{bmatrix}$$

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- Example:

$$\tau \frac{dx}{dt} = -x$$

$$M^{-1} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} - \begin{bmatrix} x_0 \\ x_0 \\ x_0 \end{bmatrix}$$

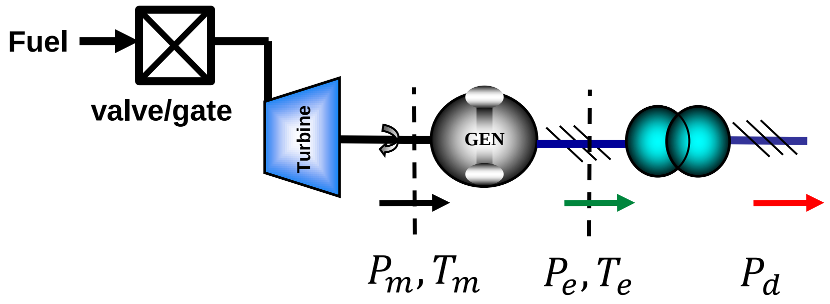
$$\tau \cdot M \cdot \begin{pmatrix} x_1 & x_0 \\ x_2 - x_0 \\ x_3 & x_0 \end{pmatrix} = -\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

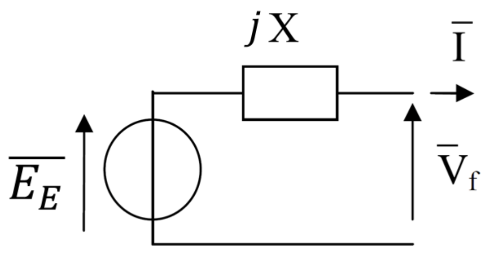
- *GOAL*: find the minimum inertia at control area level so that the frequency stability following major events (short-circuits, line trips, etc.) is guaranteed.

$$\min \sum_{i \in \boldsymbol{BUS}} H_i$$

such that

$$\left.\begin{aligned} 2H_i \frac{d\Delta\omega_i}{dt} &= P_{m,i} - P_{e,i} - D_i \cdot \Delta\omega_i \\ \frac{d\delta_i}{dt} &= \omega_s \cdot \Delta\omega_i \end{aligned}\right]$$





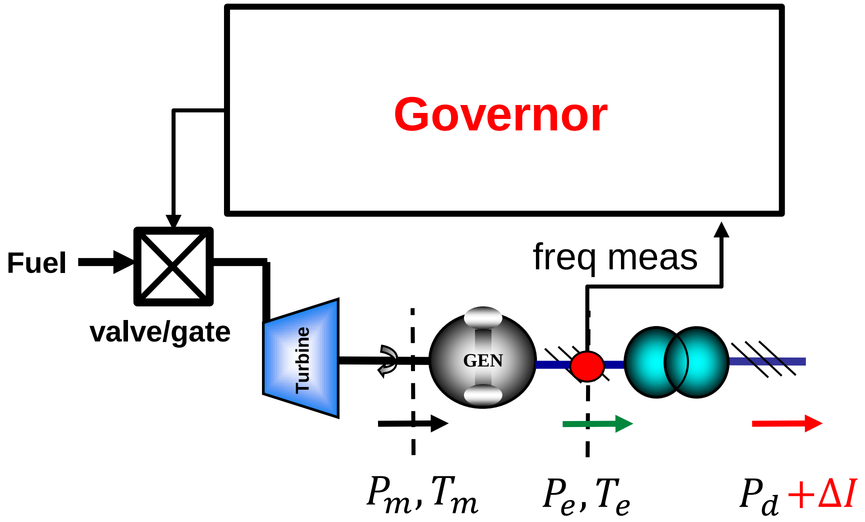
$$\overline{E_E} = E_E \cdot e^{j\delta}$$

- *GOAL*: find the minimum inertia at control area level so that the frequency stability following major events (short-circuits, line trips, etc.) is guaranteed.

$$\min \sum_{i \in \boldsymbol{BUS}} H_i$$

such that

$$\frac{dP_{mi}}{dt} = \frac{1}{T_{govi}}\left[P_{mi}^{0} + \frac{1}{R_i}\left(\frac{\Delta\omega_i}{\omega_s} - P_{mi}\right)\right]$$



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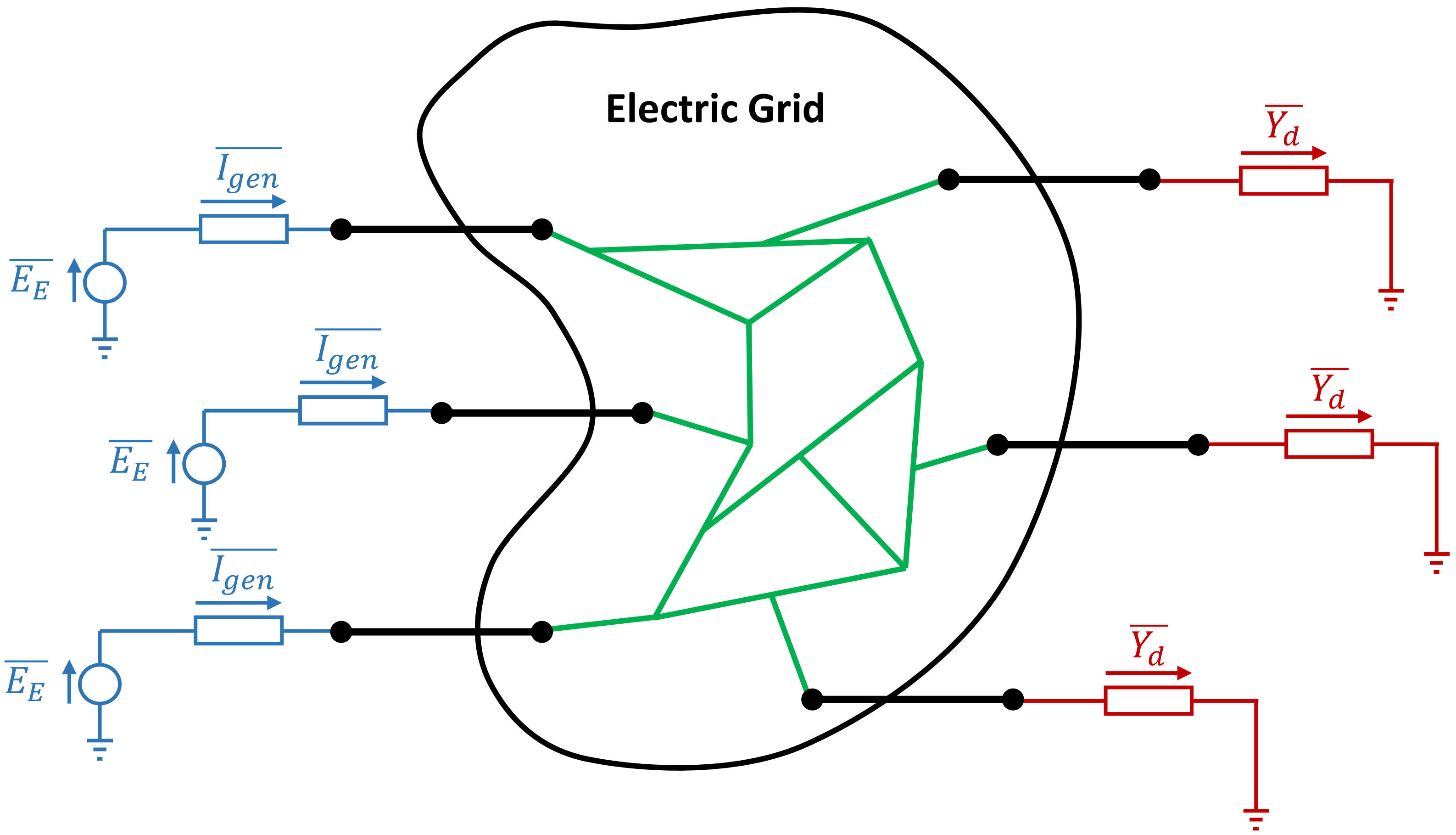
- *Grid model & algebraic link equations*:



$$[I] = [Y][V]$$

$$\Downarrow$$

$$\begin{bmatrix} I_{gen} \\ 0 \end{bmatrix} = \begin{bmatrix} Y_A & Y_B \\ Y_C & Y_D \end{bmatrix} \begin{bmatrix} E_E \\ V \end{bmatrix}$$

$$\Downarrow$$

$$I_{gen} = \left(Y_A - Y_B \cdot {Y_D}^{-1} \cdot Y_C\right) * E_E$$

$$I_{gen} = Y_{red} * E_E$$

$$\min \sum_{i \in \boldsymbol{BUS}} H_i$$

such that

$$P_{e,i}(t) = \sum_{j=1}^{ngen} \left(E_{E\,i} * E_{E\,j} * \left| Y_{red,ij} \right| * cos\left(\delta_i(t) - \delta_j(t) - \theta_{ij}\right) \right)$$

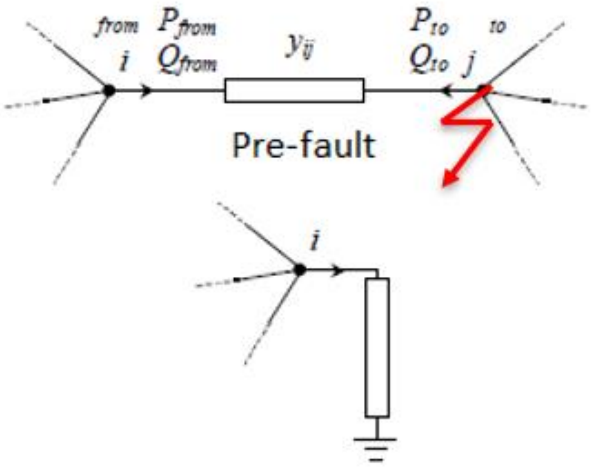
$$\left(\sum_j H_j \cdot A_{n,j} \right) \cdot \omega_a^{COI^t} = \sum_j H_j \cdot A_{n,j} \cdot \omega_j^t$$ COI frequency calculation

$$ROCOF_i^{500} \leq 2 \qquad ROCOF_i^{1000} \leq 1.5 \qquad ROCOF_i^{2000} \leq 1.25$$ Stability Constraints

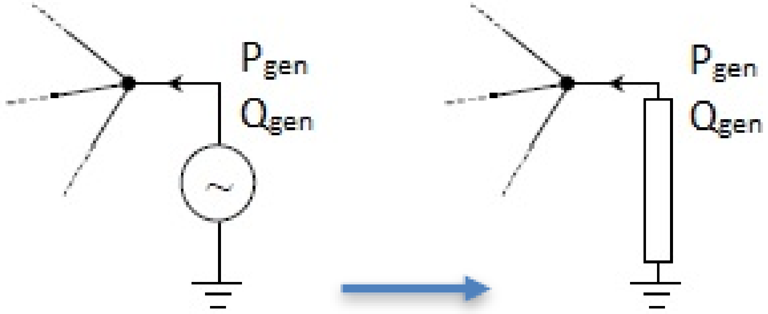
- *Modeling events*:

$$P_{e,i}(t) = \sum_{j=1}^{ngen} \left(E_{E_i} * E_{E_j} * |Y_{red,ij}| * cos\,(\delta_i(t) - \delta_j(t) - \theta_{ij}) \right)$$

Short-Circuit



Generation trip



TSC-OPF – Frequency stability model
Numerical test 1: method performance

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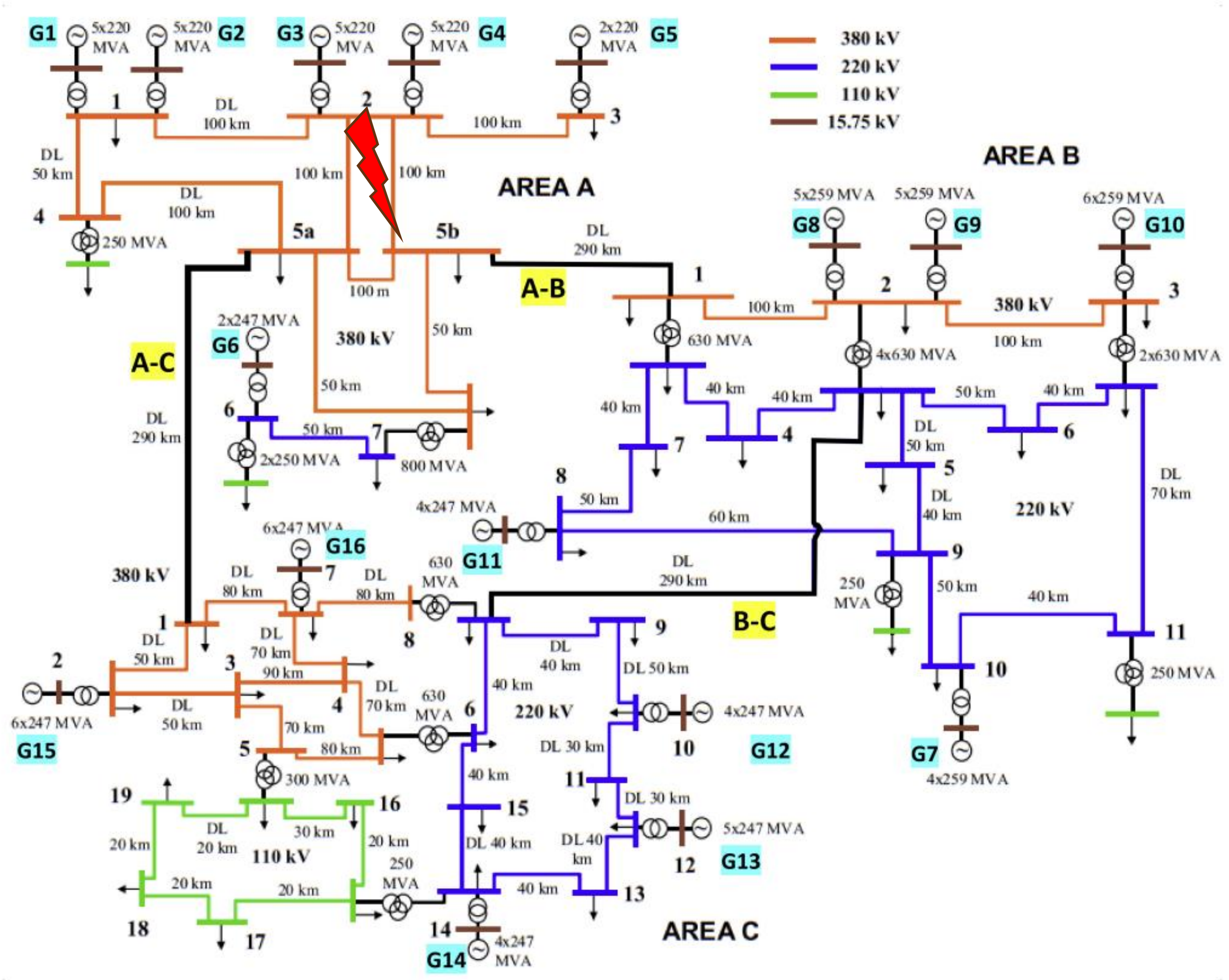
Rueda Network (a CWE equivalent):

- 16 buses with generating units
- Optimization model transient simulation results were compared to results of Digsilent, a tool for PS transient simulation
- Shortcircuit at bus A2 is applied and cleared after 200 ms



TSC-OPF – Frequency stability model
Numerical test 1: method performance

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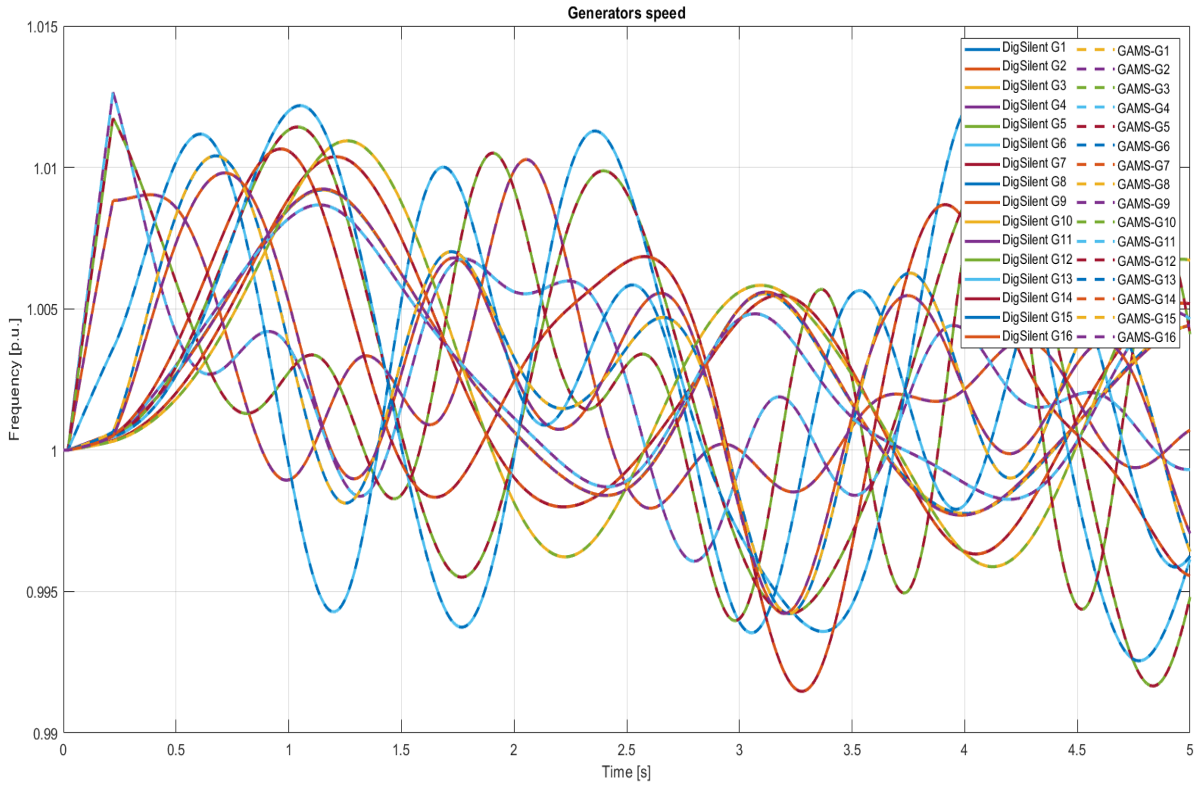
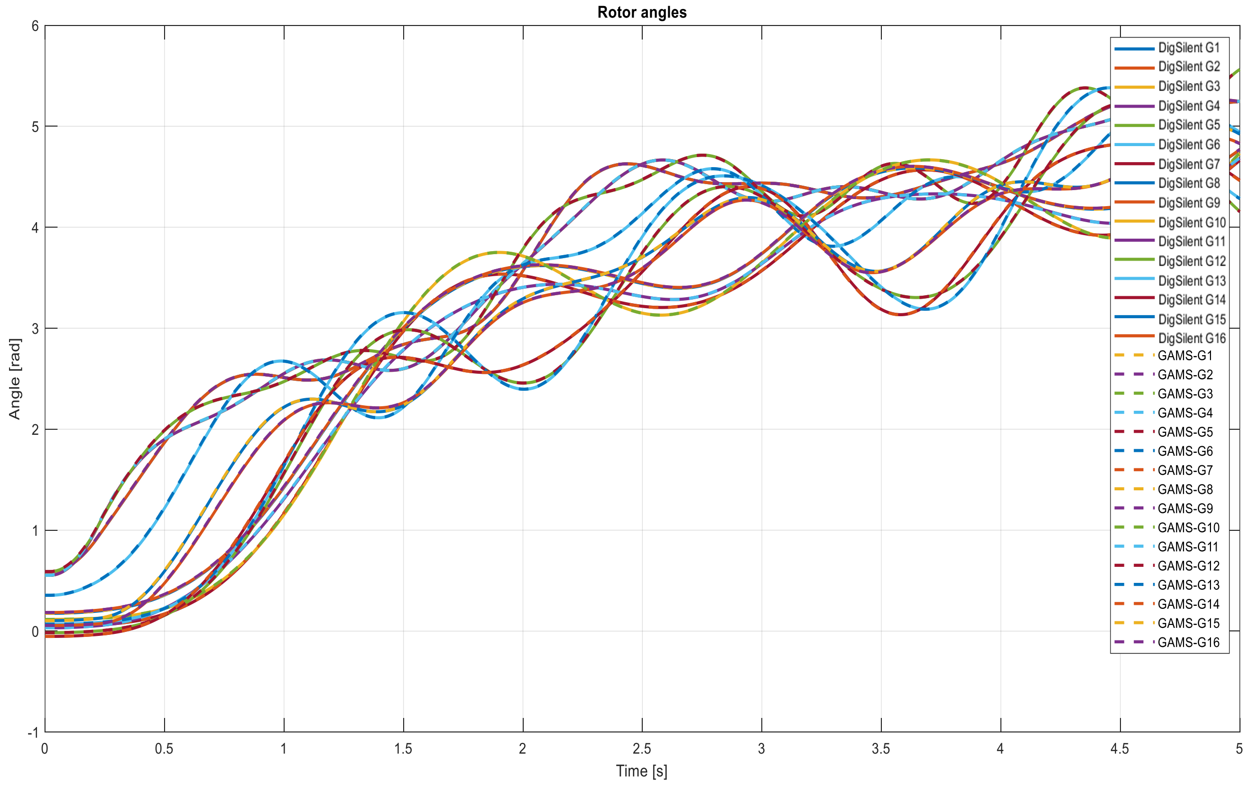
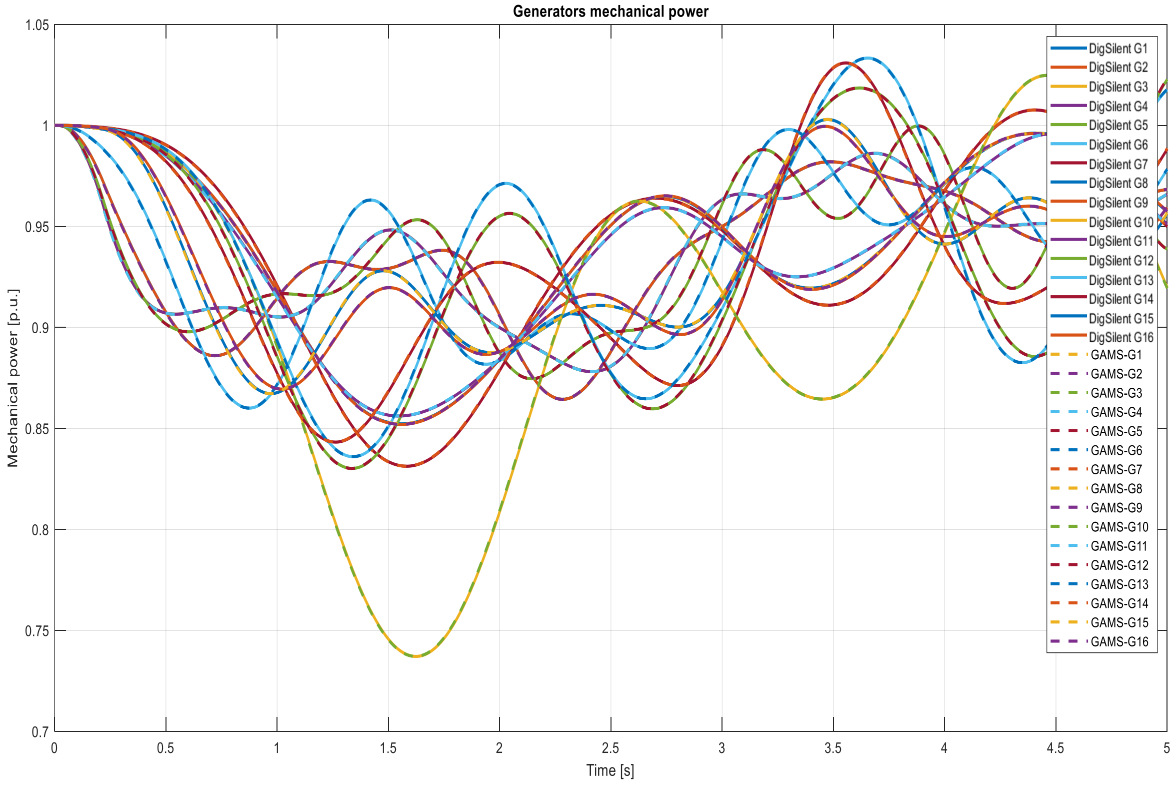




Numerical test 1: method performance



Simulation Discretization method	Δt_{max} [ms]
Trapezoid	20
Orthogonal colocation	**250**

Sicily Network:

- Analyzed scenario: 8th May 2019

- minimum inertia available: 71.5 s
- maximum inertia available: 247 s

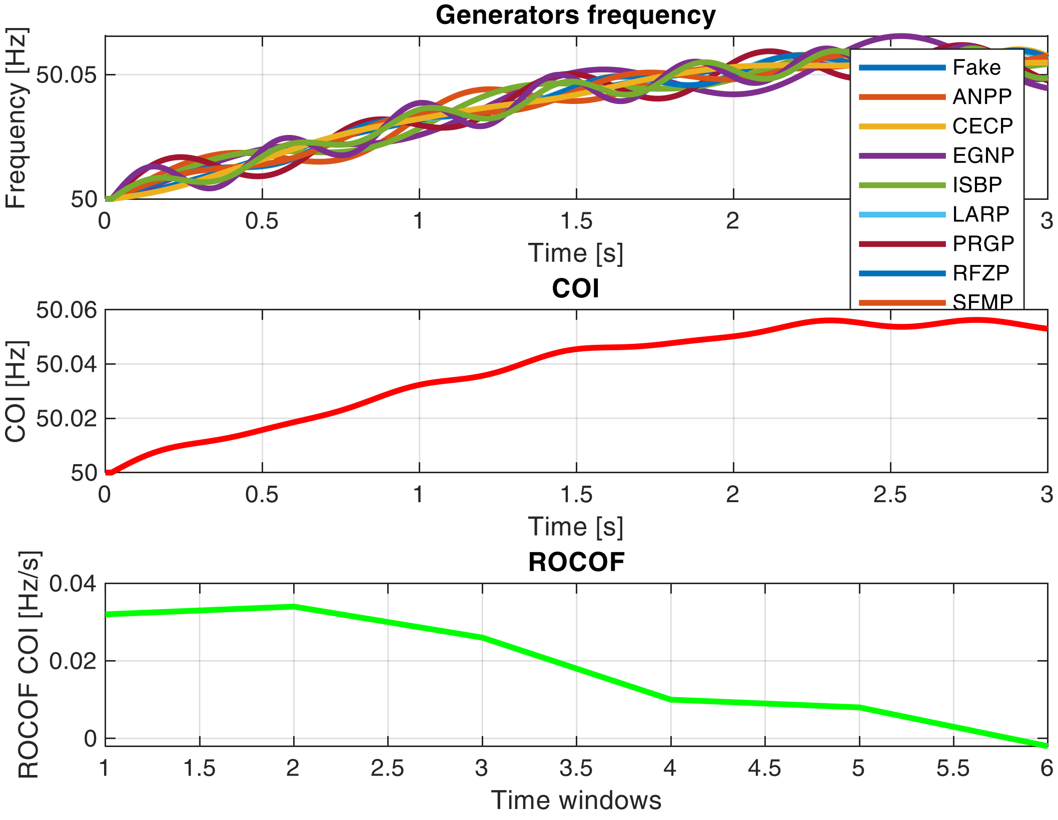
- a series of events have been considered and the minimum inertia required was determined as the envelope (max(min))

[Grid Map downloads (entsoe.eu)]

Sicily Network: 90 MW demand disconnection

- the optimizer brings the inertia to the minimum possible

- stability limit is well satisfied → not a critical case



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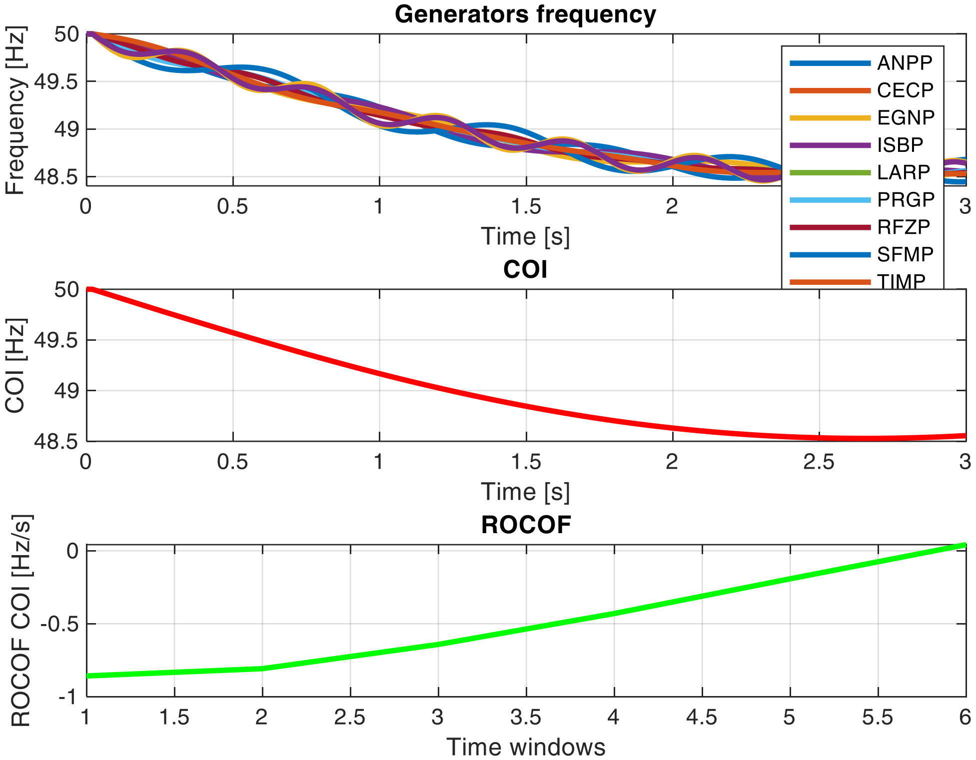
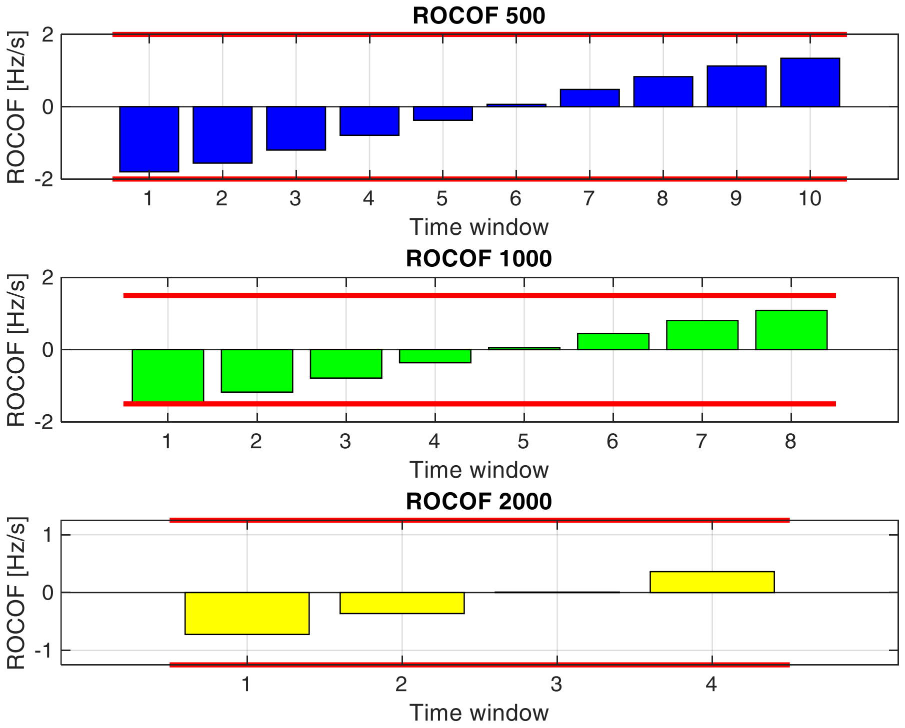
Sicily Network: Separation from continent





- the optimizer brings the inertia to the minimum possible while ROCOF1000 limit is pushed to its lower bound
- the case is critical in the minimum inertia is $H_{min} = 108\ s$

Minimum inertia constrain in ASM algorithm

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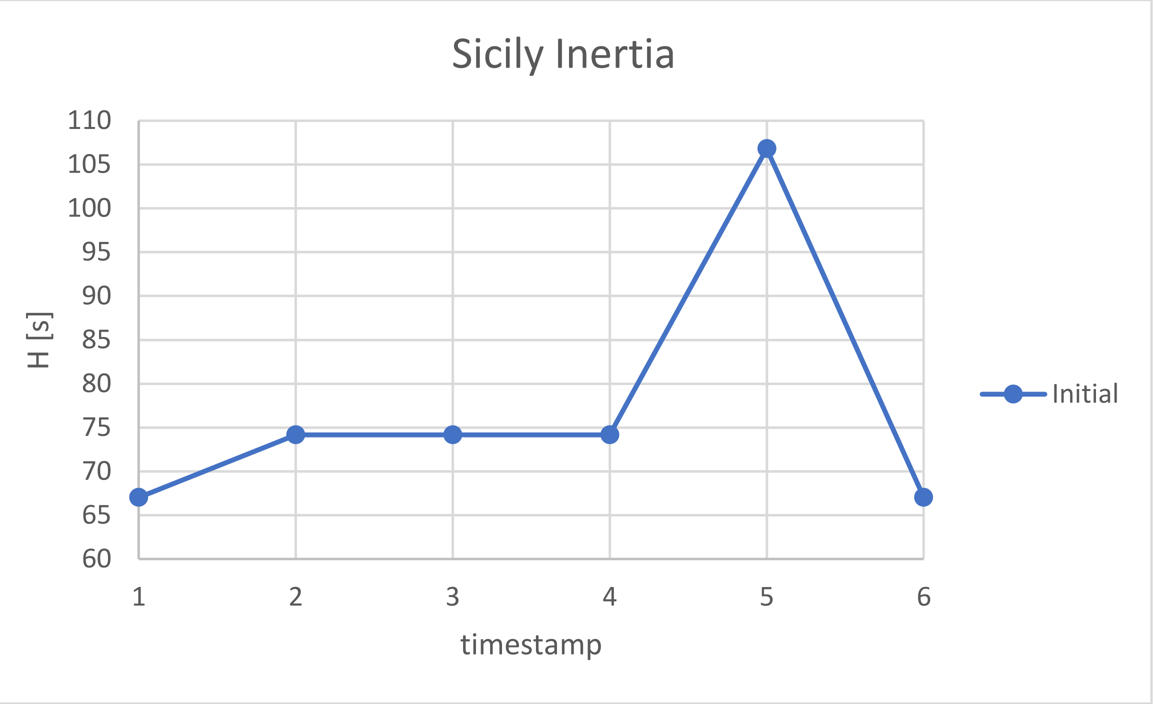

The minimum inertia requirement is not satisfied by the initial dispatch provided by the energy market. In ASM formulation a minimum zonal inertia constrain is added:

$$\sum Y_{u,t} \cdot E_u \geq \sum (Y_{u,t} \cdot A_u^n) \cdot H_{min,t}$$

where:

- H_u is the inertia of generating unit u;
- E_u is the kinetic energy of generating unit u;
- A_u^n is the apparent nominal power of generating unit u;
- $H_{min,t}$ is the minimum level of inertia required.

Future Work

- test other "orthogonal collocation" approximation functions for x(t) that could allow for a larger Δt

- consider also converter-based DERs dynamic models with synthetic inertia capabilities

Questions