# Regularized Benders Decomposition for High Performance Capacity Expansion Models


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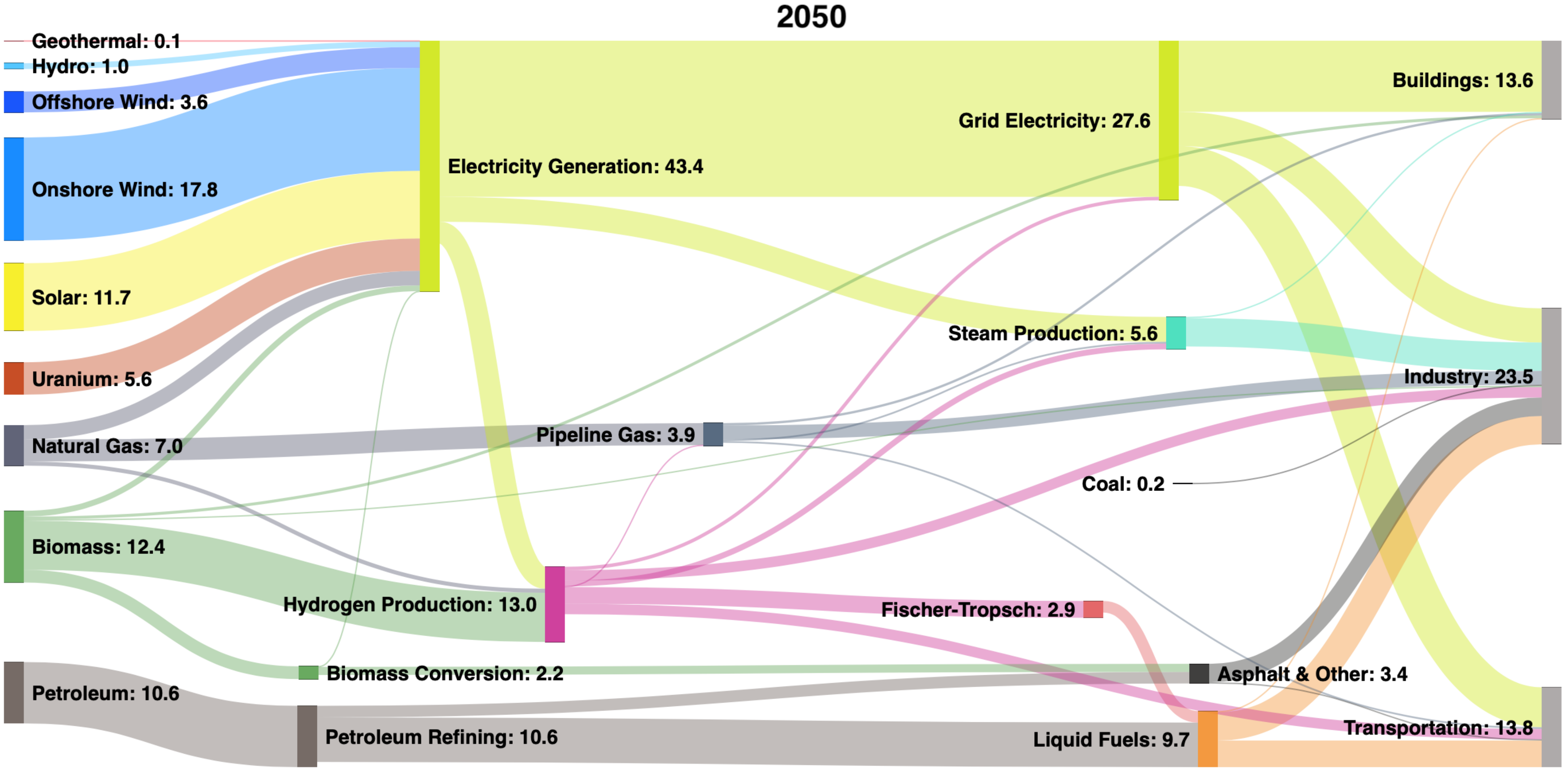
PRINCETON UNIVERSITY

andlinger center for energy+the environment

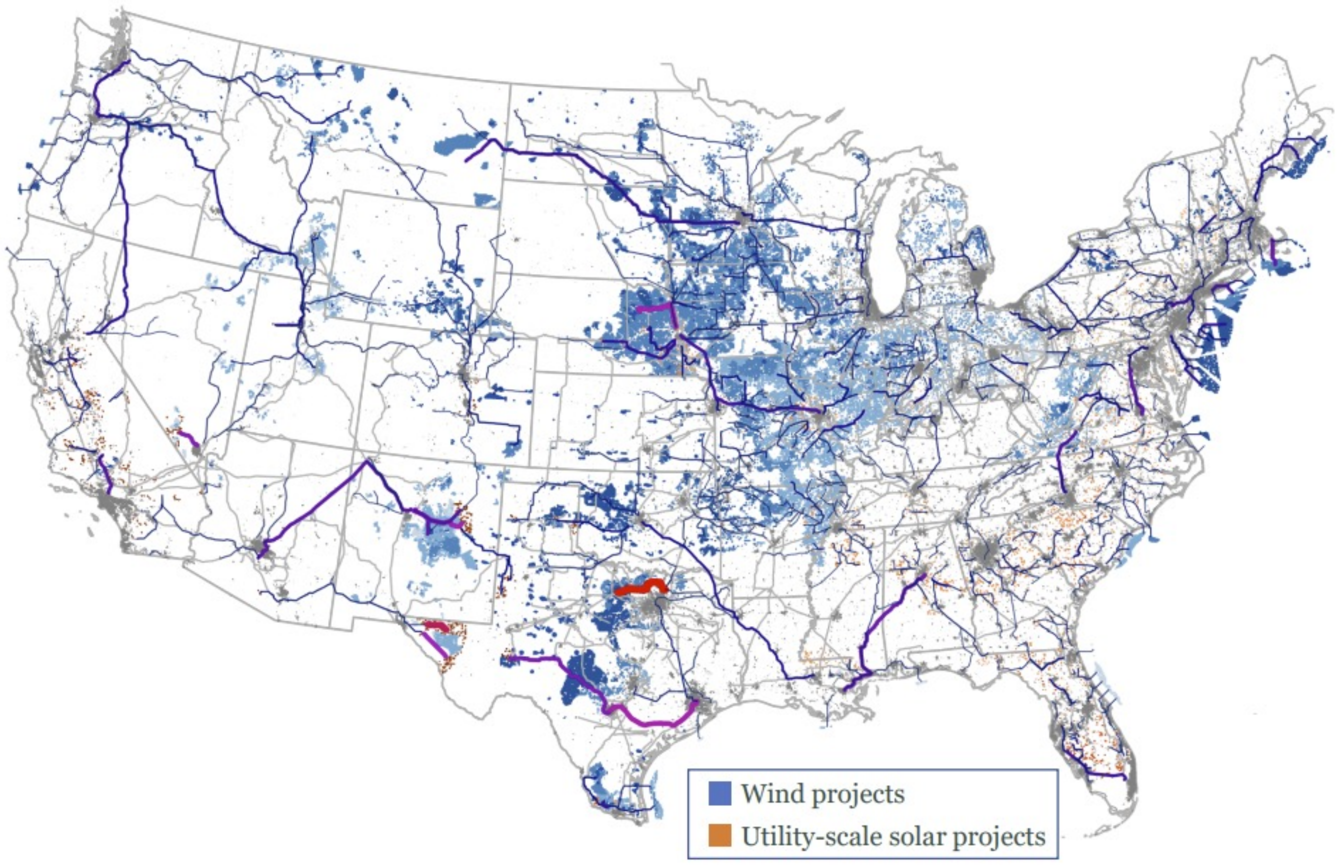
# The electricity sector will play a major role in the journey to decarbonize our economy




Projected US primary energy flows for a net-zero scenario. Source: https://netzeroamerica.princeton.edu/

# Capacity expansion models are key to support a sustainable and equitable transition in the electricity sector

- Utility integrated resource and investment planning.
- Techno-economic analysis of emerging energy technologies.
- Evaluation of system-level impacts of policy and regulatory frameworks.
- Uncertainty and scenario analysis.
- Robust strategies to transition to net-zero emissions electricity systems.




Source: https://netzeroamerica.princeton.edu/

# Large-scale Mixed Integer Linear Problems (MILPs)

## INVESTMENT DECISIONS

### Capacity Variables

- How much new capacity for wind farms / solar panels/ batteries / electrolyzers?
- Where to install them?
- How much coal and natural gas capacity should be retired?

## OPERATIONAL AND POLICY CONTRAINTS

### Linear Constraints

Engineering detail:

- Unit commitment
- Economic dispatch
- Hourly temporal resolution
- Linearised power flow

Policy constraints:

- $CO_2$ emissions cap

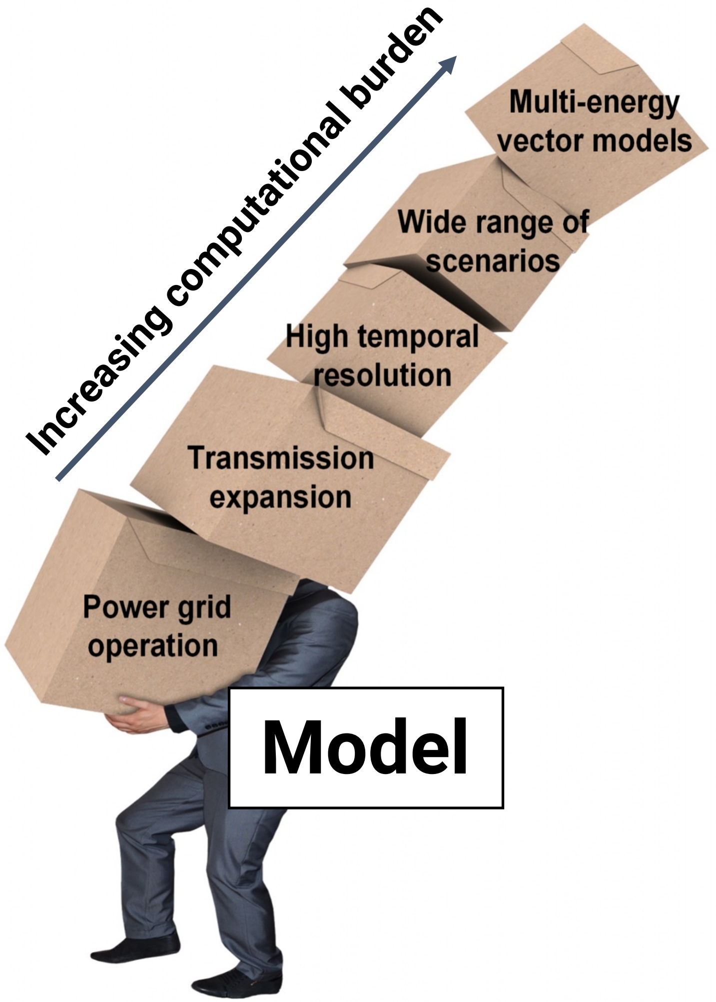
## MODELING REQUIREMENTS

### High Dimensionality

- Long term planning horizon
- Multiple scenarios for robustness
- High spatial resolution (e.g., to evaluate local socio-economical effects)

**Minimize investment and operating costs**

# The need: advancing solution algorithms for improved computational performance




Full-resolution MILP with O(100,000,000s) variables and constraints, which is intractable even using the best commercial solvers.

In practice, modelers rely on carefully designed abstraction techniques:

- Sampling representative time periods or ignoring sequential operations entirely.
- Aggregating regions into larger geographical zones.
- Ignoring key operational constraints.

These abstractions can ensure models are computationally tractable but come at the cost of **significantly reduced accuracy** that impacts their ability to provide credible decision support.

# Some exciting progress

- **New computationally efficient decomposition** methods for electricity capacity expansion models.
- **Runtime scales linearly** with operational sub-periods (e.g., weeks), enabling planning with multiple weather years or planning stages with 8,736 hours
- **Discrete investment/retirement decisions** capture economies of unit scale (e.g., transmission lines)
- Linearized unit commitment decisions in operations at hourly time steps
- Long-duration energy storage & reservoir hydro

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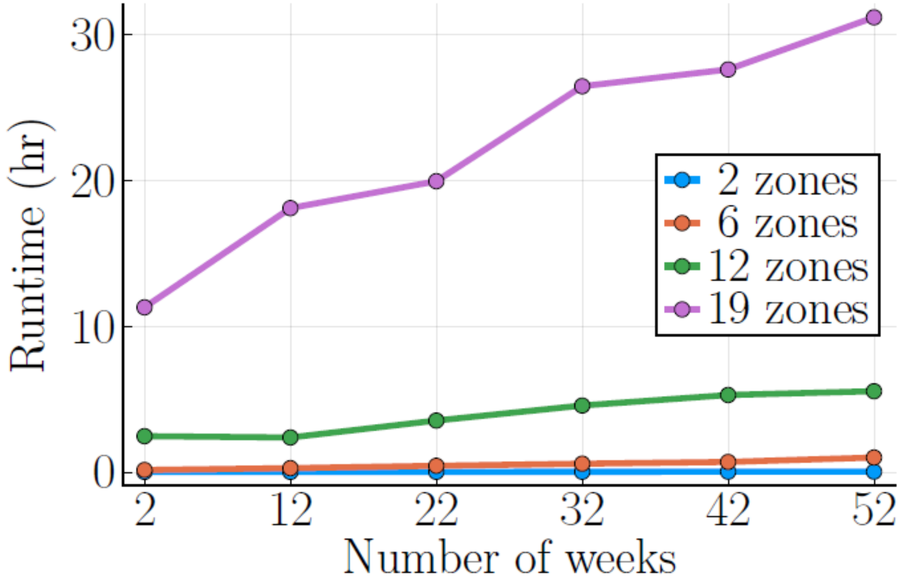
**A Computationally Efficient Benders Decomposition for Energy Systems Planning Problems with Detailed Operations and Time-Coupling Constraints**

Anna Jacobson,[a,*] Filippo Pecci,[b] Nestor Sepulveda,[c,d] Qingyu Xu,[e] Jesse Jenkins[f]

https://doi.org/10.1287/ijoo.2023.0005




New preprint: "Regularized Benders Decomposition for High Performance Capacity Expansion Models" available at: https://arxiv.org/abs/2403.02559

# How is it done?

# Mathematical formulation

$$\min \sum_{p\in P}\left(f_p^T y_p + \sum_{w\in W_p} c_w^T x_w\right)$$

$$\text{s.t.} \quad A_w x_w + B_w y_p \le b_w, \quad \forall w \in W_p,\ p \in P$$

$$\sum_{w\in W_p} Q_w x_w \le e_p, \quad \forall p \in P$$

$$\sum_{p\in P} R_p y_p \le r$$

$$x_w \ge 0, \quad \forall w \in W_p,\ \forall p \in P$$

$$y_p \in \mathbb{Z}^m, \quad \forall p \in P$$

- Objective: minimize investment and operating costs
- Planning decision variables: discrete investment and retirement decisions for each planning period.
- Operational decision variables: generators dispatch, storage levels, unit commitment, power flows.
- Linearized power flow, unit commitment, and short-duration energy storage constraints with cyclic approximation (linking first and last hour of each sub-period).
- Sub-period coupling constraints, including policy constraints and long-duration storage constraints (e.g. hydropower resources)

# Mathematical formulation

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Common to many existing capacity expansion models, including widely used open-source models like PyPSA, SWITCH, and GenX.

- Objective: minimize investment and operating costs
- Planning decision variables: discrete investment and retirement decisions for each planning period.
- Operational decision variables: generators dispatch, storage levels, unit commitment, power flows.
- Linearized power flow, unit commitment, and short-duration energy storage constraints with cyclic approximation (linking first and last hour of each sub-period).
- Sub-period coupling constraints, including policy constraints and long-duration storage constraints (e.g. hydropower resources)

# Block-structured capacity expansion model

$$\min \sum_{p \in P} \left( f_p^T y_p + \sum_{w \in W_p} c_w^T x_w \right)$$

$$\text{s.t.} \quad A_w x_w + B_w y_p \le b_w, \quad \forall w \in W_p,\ p \in P$$

$$\sum_{w \in W_p} Q_w x_w \le e_p, \quad \forall p \in P$$

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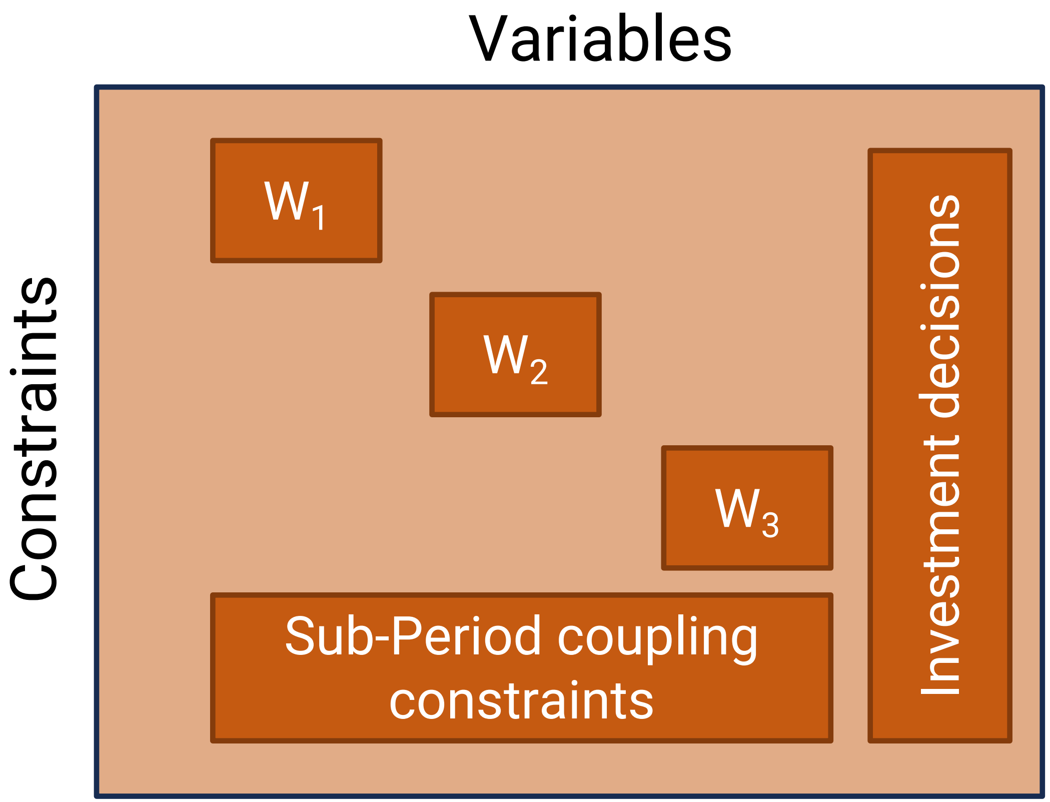
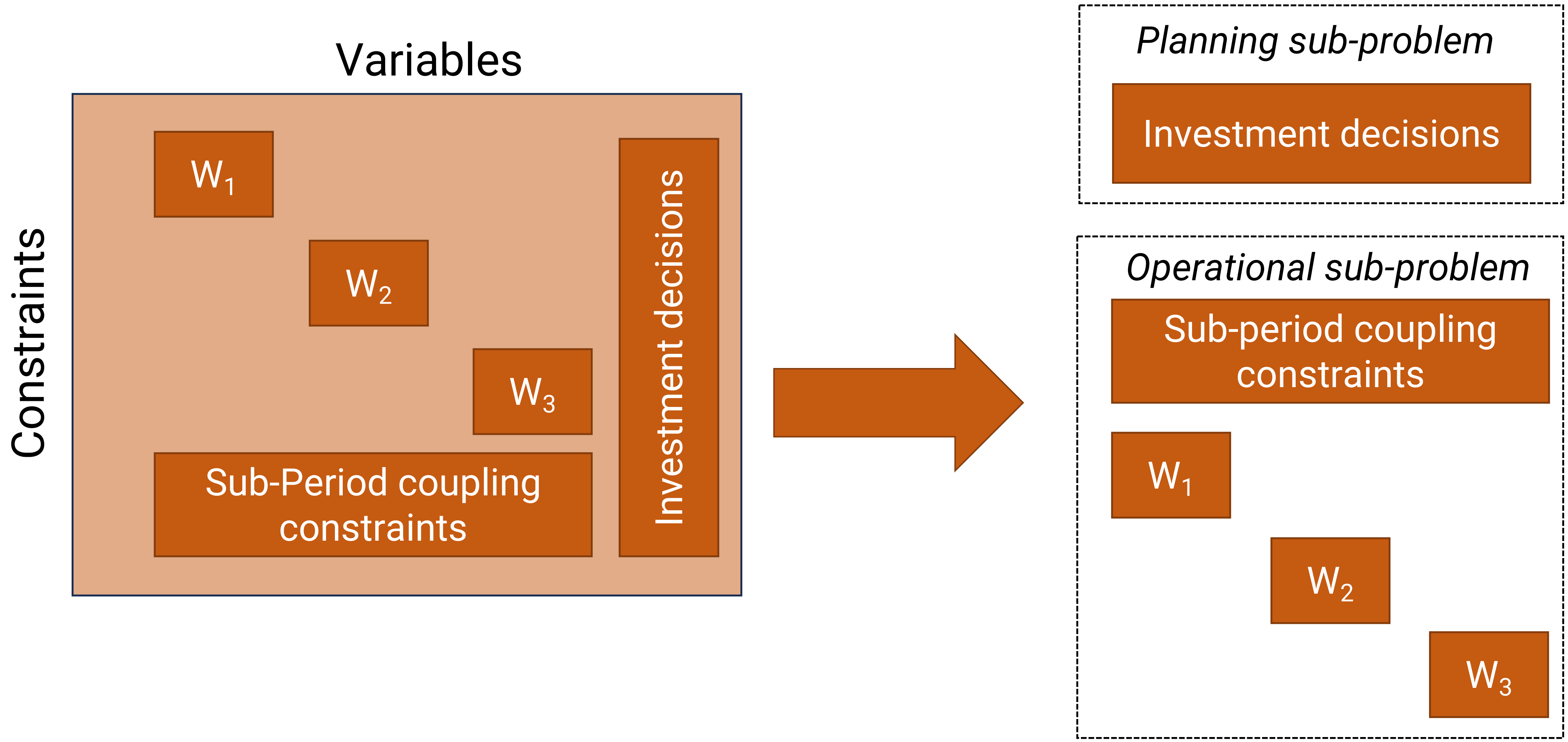


# Standard Benders decomposition method



# Standard Benders decomposition

Iterate between planning and operational sub-problems

**Planning sub-problem**

$$
\begin{aligned}
\min \quad & \sum_{p \in P} \left( f_p^T y_p + \theta_p \right) \\
\text{s.t.} \quad & \theta_p \geq g_p(y_p^j) + (y - y_p^j)^T \pi_p^j, \quad j = 1, \ldots, k, \ \forall p \in P \\
& \sum_{p \in P} R_p y_p \leq r \\
& y_p \in \mathbb{Z}^m, \quad \forall p \in P
\end{aligned}
$$

**Operational sub-problem**

$$
\begin{aligned}
g_p(y_p^{k+1}) = \min \quad & \sum_{w \in W_p} c_w^T x_w \\
\text{s.t.} \quad & A_w x_w + B_w y_p \leq b_w, \quad \forall w \in W_p \\
& \sum_{w \in W_p} Q_w x_w \leq e_p \\
& y_p = y_p^{k+1} \quad (\pi_p^{k+1}) \\
& x_w \geq 0, \quad \forall w \in W_p
\end{aligned}
$$

Note that the operational sub-problem represents a full year with hourly resolution

# Decoupling operational sub-periods

Using auxiliary variables, we subdivide the operational year into shorter sub-periods:

- In the case of a $CO_2$ emission cap, we have that:

$$\sum_{w \in W_p} q_w^T x_w \leq \epsilon^{\mathrm{CO_2}} \quad \text{is equivalent to:} \quad \begin{cases} q_w^T x_w \leq z_w^b \\ \sum_{w \in W_p} z_w^b = \epsilon^{\mathrm{CO_2}} \end{cases}$$

  where variables $z_w^b$ are emission budgets for each sub-period.

- In the case of long-duration energy storage, we introduce auxiliary variables $z_w^{\mathrm{start}}$ and $z_w^{\mathrm{end}}$ corresponding to storage levels at the start and end of each sub-period.

# Decoupling operational sub-periods

With these reformulations, we can apply Bender decomposition considering investment and retirement decisions as well as sub-period decoupling variables as planning decisions

$$
\begin{aligned}
\min \quad & \sum_{p \in P} \left( f_p^T y_p + \sum_{w \in W_p} c_w^T x_w \right) \\
\text{s.t.} \quad & A_w x_w + B_w y_p \leq b_w, \quad \forall w \in W_p,\ p \in P \\
& Q_w x_w \leq z_w, \quad \forall w \in W_p,\ p \in P \\
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\end{aligned}
$$

# New Benders decomposition scheme

Planning sub-problem

$$
\begin{aligned}
\min\ & \sum_{p\in P}\left(f_p^T y_p + \sum_{w\in W_p}\theta_w\right)\\
\text{s.t.}\ & \theta_w \geq g_w(y_p^j) + (y - y_p^j)^T \pi_w^j + (z_w - z_w^j)^T \mu_w^j,\\
& j = 1,\dots,k,\ \forall w \in W_p,\ p\in P\\
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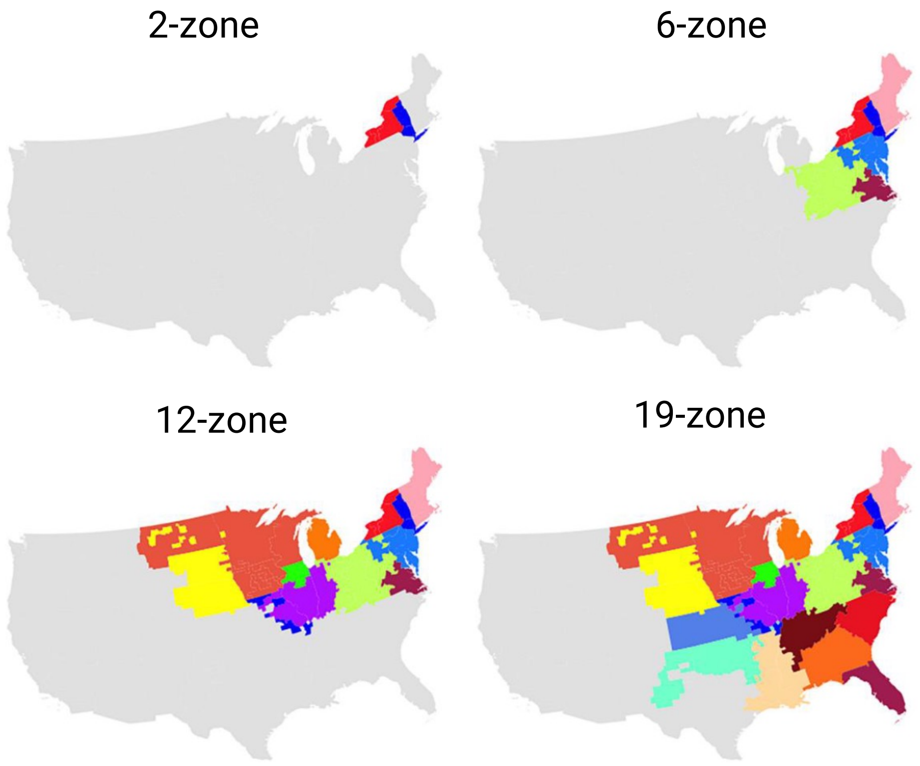
Operational sub-problems

$$
\begin{aligned}
g_w(y_p^{k+1}) = \min\ & c_w^T x_w\\
\text{s.t.}\ & A_w x_w + B_w y_p \leq b_w\\
& Q_w x_w \leq z_w\\
& y_p = y_p^{k+1} \quad (\pi_w^{k+1})\\
& z_w = z_w^{k+1} \quad (\mu_w^{k+1})\\
& x_w \geq 0, \quad \forall w\in W_p
\end{aligned}
$$

**The operational sub-periods can now be optimized in parallel**

OK but…does it work?

# Test cases



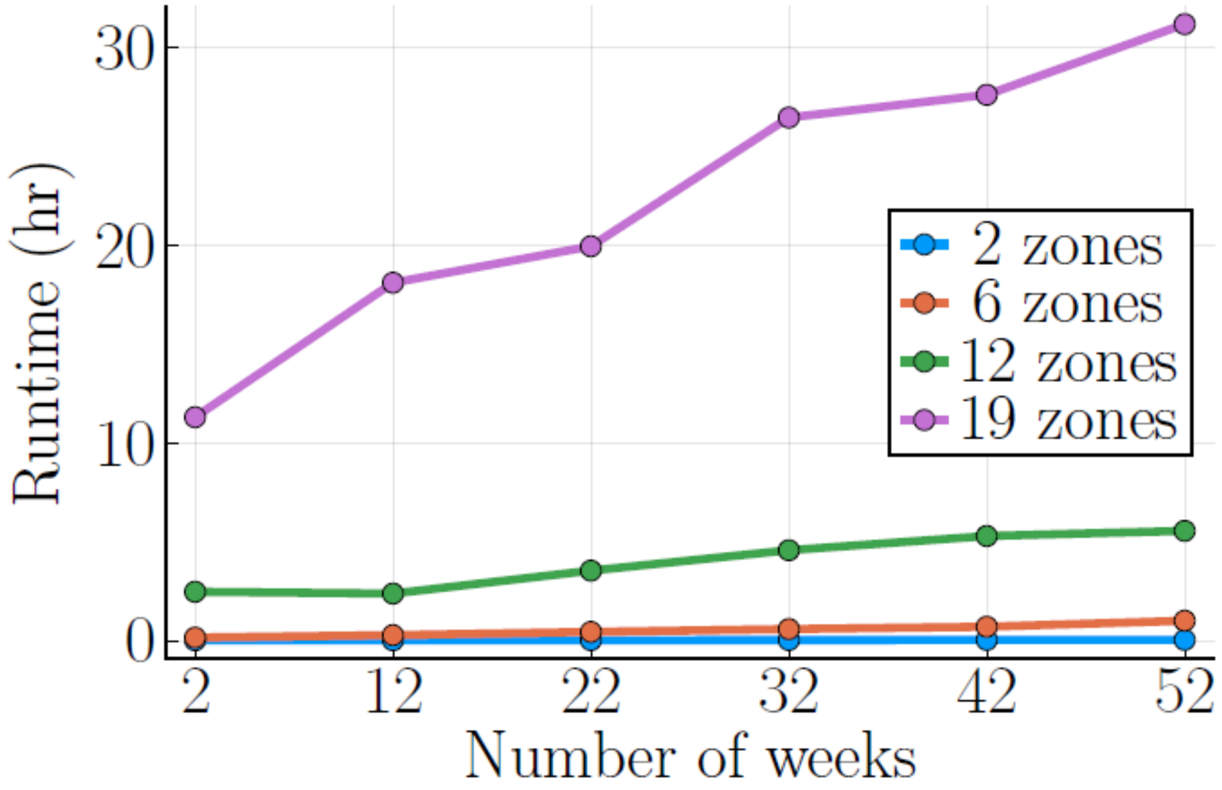

| Zones | Number of resource clusters | Variables | Constraints |
|---|---|---|---|
| 2 | 62 | 1.1 million | 3.4 million |
| 6 | 175 | 3.4 million | 10.5 million |
| 12 | 285 | 6.2 million | 19.3 million |
| 19 | 437 | 9.7 million | 30.4 million |

- Single-period planning model with **hourly resolution**.
- These model sizes are for a case with 52 weeks, i.e., 8736 time steps.
- Generation and transmission expansion modelled by **integer investment decisions.**
- We considered cases with CO2 policy constraints.

# Decomposed models can optimize integer investments

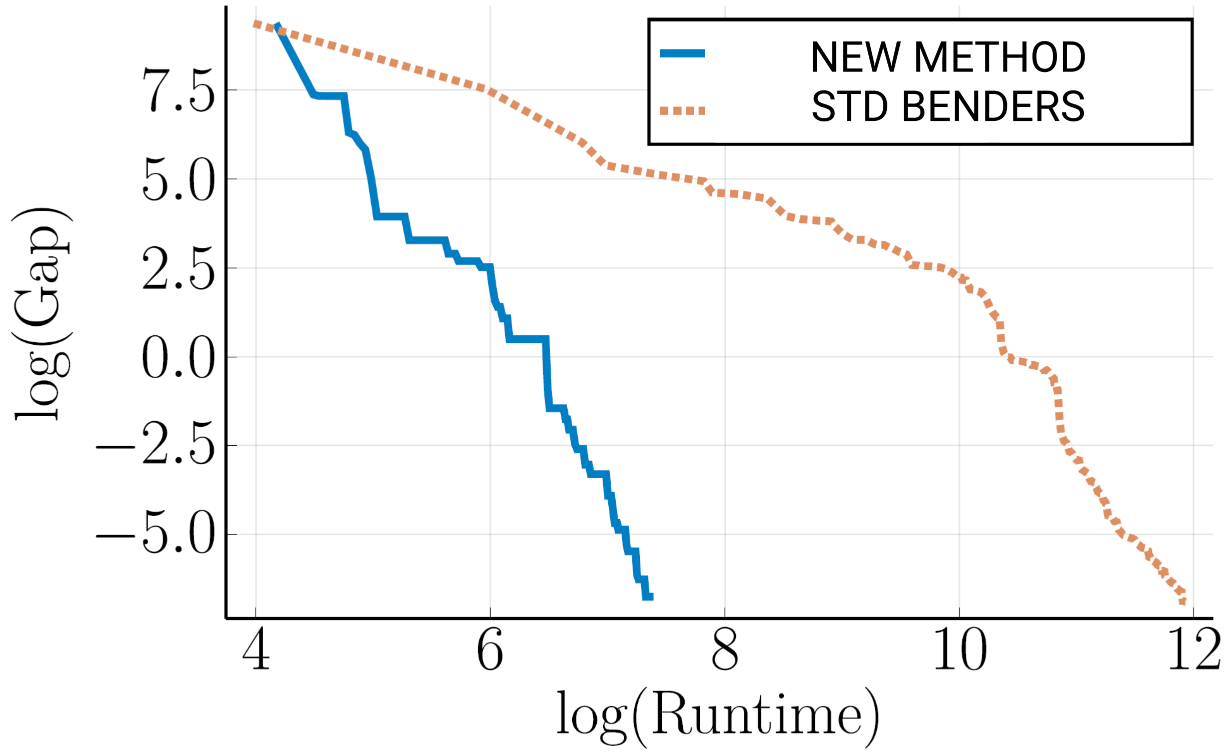
| | Zones \ Weeks | 2 | 12 | 22 | 32 | 42 | 52 | |
|---|---|---|---|---|---|---|---|---|
| **Decomposition Runtime (100s)** | 2 | **1.1** | **1.2** | **1.4** | **1.6** | **1.9** | **2.0** | |
| | 6 | **6.1** | **10.6** | **16.4** | **21.6** | **25.9** | **36.8** | |
| | 12 | 89.5 | **86.2** | **128.0** | **165.2** | **191.1** | **200.4** | |
| | 19 | 407.5 | **652.4** | **718.9** | **953.3** | **994.7** | **1123.4** | **~30 hours** |
| **Monolithic Runtime (100s)** | 2 | 2.9 | 58.6 | 112.5 | 702.8 | 1344.3 | 651.9 | |
| | 6 | 7.5 | 129.4 | TimeOut | TimeOut | TimeOut | TimeOut | |
| | 12 | 24.5 | TimeOut | TimeOut | TimeOut | TimeOut | TimeOut | |
| | 19 | 252.5 | TimeOut | TimeOut | TimeOut | TimeOut | TimeOut | |
| Ratio | 2 | 2.5 | 48.0 | 78.2 | 444.8 | 722.7 | 329.2 | |
| | 6 | 1.2 | 12.3 | - | - | - | - | |
| | 12 | 0.3 | - | - | - | - | - | |
| | 19 | 0.6 | - | - | - | - | - | |

# Runtime scales linearly with number of weeks



# Outperform standard Benders implementation

Results for the 6-zone case with 22 weeks (the largest that the standard decomposition method could solve in 48 hours)



# Further progress through regularization

- The planning sub-problem guesses optimal planning decisions based on estimated costs of system's operations.

- Especially at early iterations, some guesses correspond to extreme planning decisions that slow down convergence.

- We **correct the guesses proposed by the planning sub-problem**, accounting for the fact that the planning sub-problem has incomplete information on system's operations (well-known idea in mathematical optimization, it includes proximal bundle methods, level-set methods…)

- We also ignore integrality constraints on investment decisions until we are sufficiently close to convergence, and **then switch them on to compute the final solution.**

# Level-set regularized Benders decomposition

$$\begin{aligned}\min\ & \sum_{p\in P}\Big(f_p^T y_p + \sum_{w\in W_p}\theta_w\Big)\\ \text{s.t.}\ & \theta_w \ge g_w(y_p^j) + (y-y_p^j)^T\pi_w^j + (z_w - z_w^j)^T\mu_w^j,\\ & j=1,\dots,k,\ \forall w\in W_p,\ p\in P\\ & \sum_{w\in W_p} D_w z_w = e_p \quad \forall w\in W_p,\ p\in P\\ & \sum_{p\in P} R_p y_p \le r\\ & y_p\in\mathbb{Z}^m, \quad \forall p\in P\\ & z_w \ge 0 \quad \forall w\in W_p,\ p\in P\end{aligned}$$

The level-set constraint

$$\begin{aligned}\min\ & R(y,z)\\ \text{s.t.}\ & \theta_w \ge g_w(y_p^j) + (y-y_p^j)^T\pi_w^j + (z_w - z_w^j)^T\mu_w^j,\\ & j=1,\dots,k,\ \forall w\in W_p,\ p\in P\\ & \sum_{p\in P}\Big(f_p^T y_p + \sum_{w\in W_p}\theta_w\Big) \le L^k + \alpha(U^k - L^k)\\ & \sum_{w\in W_p} D_w z_w = e_p \quad \forall w\in W_p,\ p\in P\\ & \sum_{p\in P} R_p y_p \le r\\ & y_p\in\mathbb{Z}^m, \quad \forall p\in P\\ & z_w \ge 0 \quad \forall w\in W_p,\ p\in P\end{aligned}$$

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Possible choices for the regularization function include:

$$R^{\ell_2}(y,z) = \|y-y^{k+1}\|_2^2 + \|z-z^{k+1}\|_2^2$$
$$R^{\ell_1}(y,z) = \|y-y^{k+1}\|_1 + \|z-z^{k+1}\|_1$$
$$R^{\ell_\infty}(y,z) = \|y-y^{k+1}\|_\infty + \|z-z^{k+1}\|_\infty$$
$$R^{\text{int}}(y,z) = 0$$

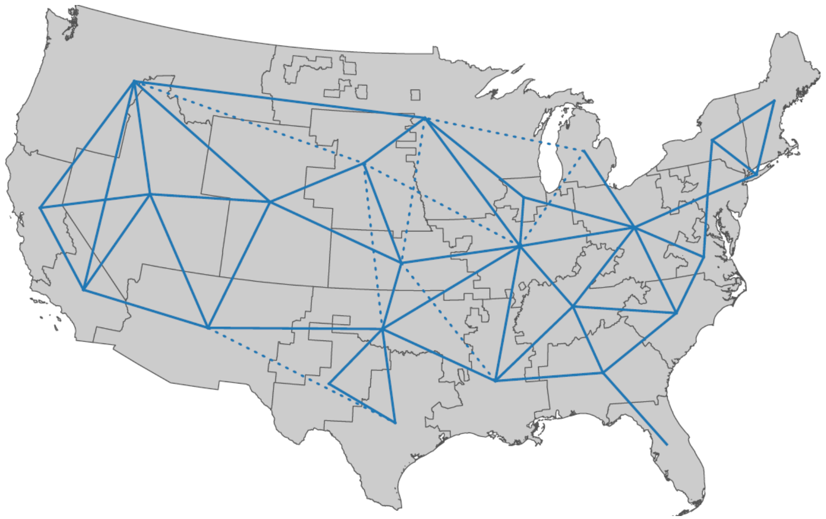
# Level-set regularized Benders decomposition with discrete planning decisions

**Step 1.** Apply regularized Benders decomposition for some choice of convex function R(.), ignoring integrality constraints on planning variables. When the convergence tolerance is satisfied, go to Step 2.

**Step 2.** Initialize the planning sub-problem with all the Benders cuts computed at Step 1, reinclude all integrality constraints, and run Benders without regularization step.

We avoid solving two MILPs at every iteration of the regularized Benders algorithm and take advantage of the (hopefully) good quality cuts computed through regularization to warm-start the planning sub-problem when integer constraints are included.

# 26-zone system for Continental US (CONUS)



**GenX model with more than <u>26,000 time steps, 70 million variables and 144 million constraints</u>**

- 3 planning stages with foresight
- 52 weeks (8736 hours) per stage
- 26 zones, over 1000 resource clusters, and 49 transmission paths

For each of the 57 inter-zonal connections, we consider 3 line voltages (230kV, 345kV, 500kV) with either single or double circuits, and a 500kV HVDC line, resulting in 7 different line classes.

Discrete transmission expansion decisions correspond to the number of new lines from each class.

Discrete generation investment or retirement decisions model the number of units in each generator and storage cluster.

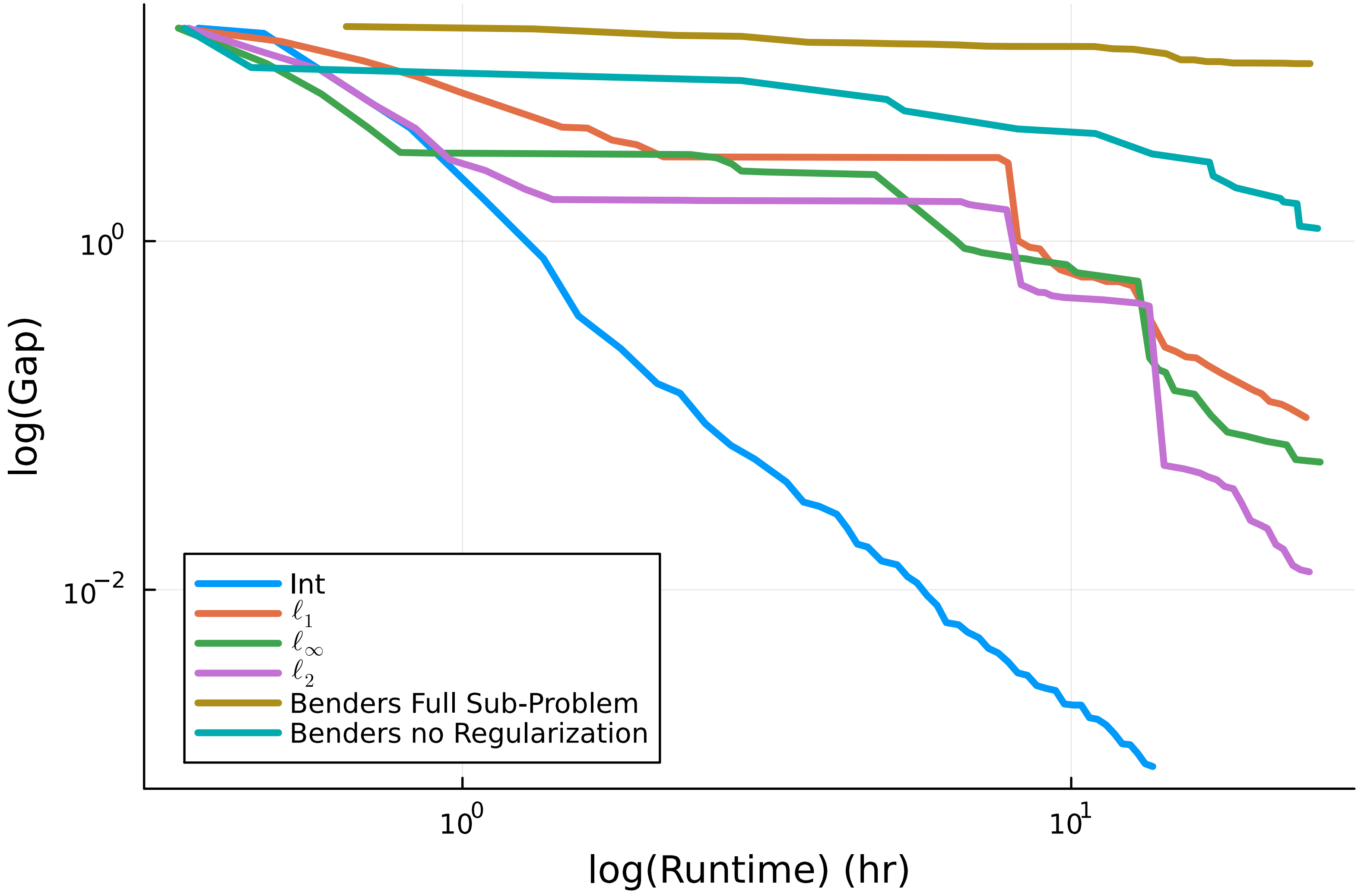
$CO_2$ cap policy constraints

Benders decomposition implemented as solver routine within the open-source capacity expansion model model GenX.

GenX.jl

# Benchmarking with continuous investment decisions

*Model solved using a single computing node with 52 cores. All level-set regularization schemes reported here have $\alpha$=0.5*



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*Model solved using a single computing node with 52 cores. All level-set regularization schemes reported here have α=0.5*

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& \sum_{p \in P} \left( f_p^T y_p + \sum_{w \in W_p} \theta_w \right) \leq L^k + \alpha (U^k - L^k) \\
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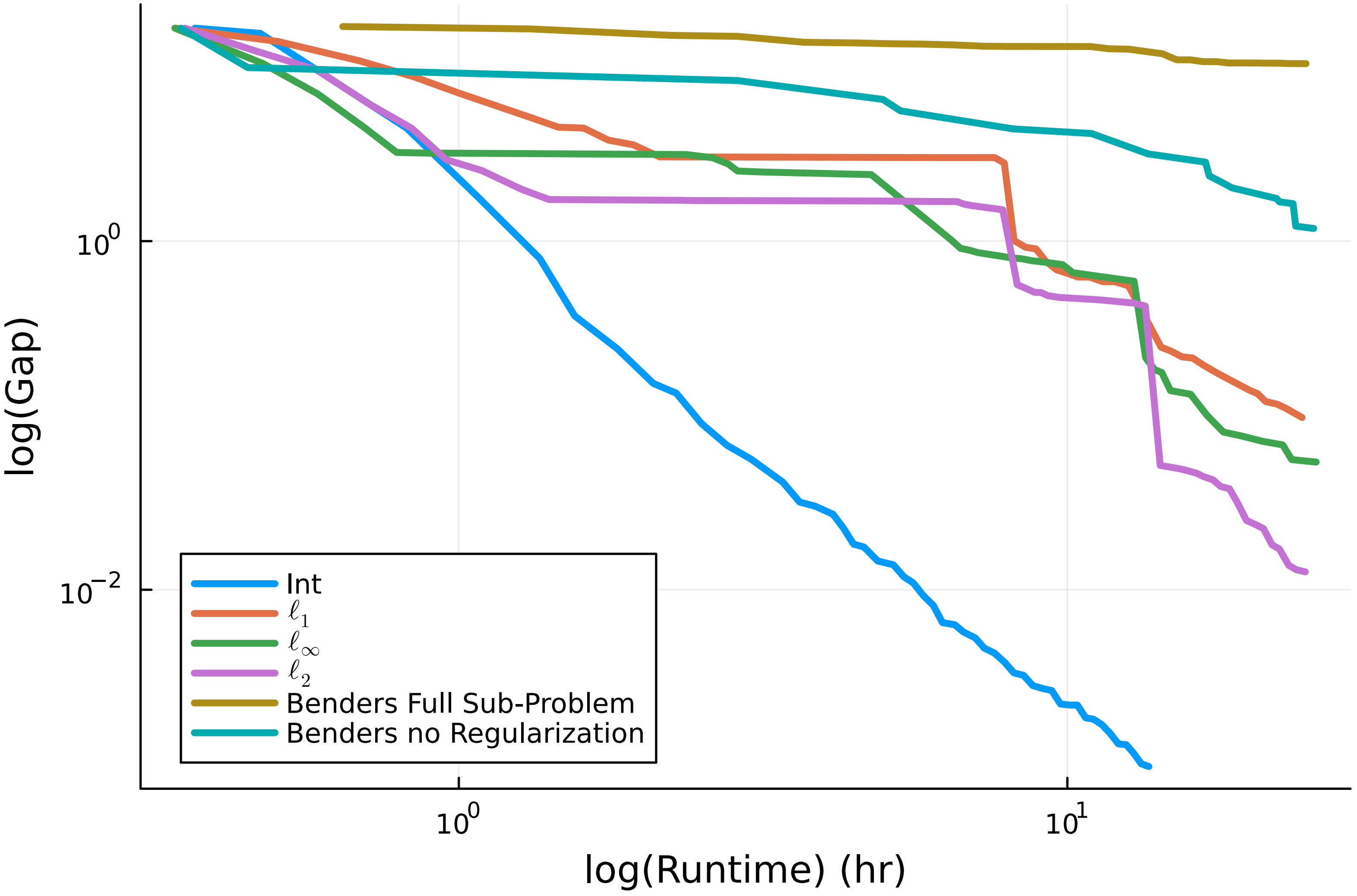
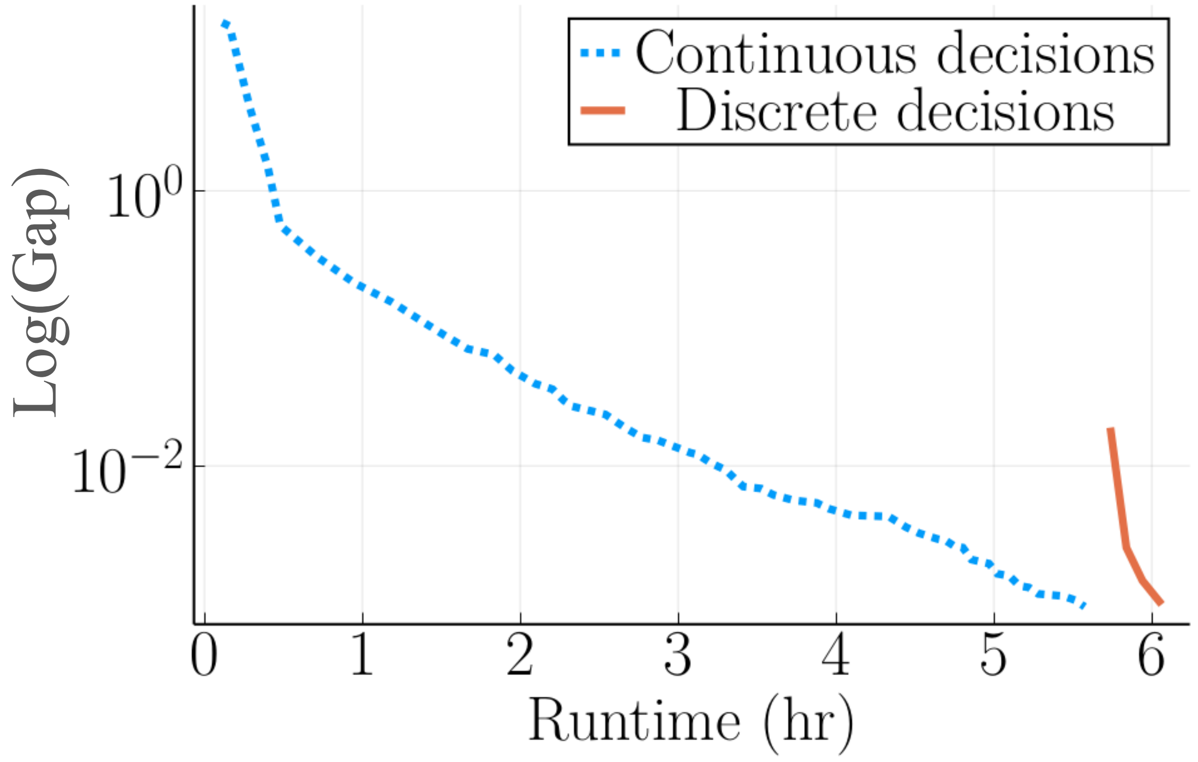


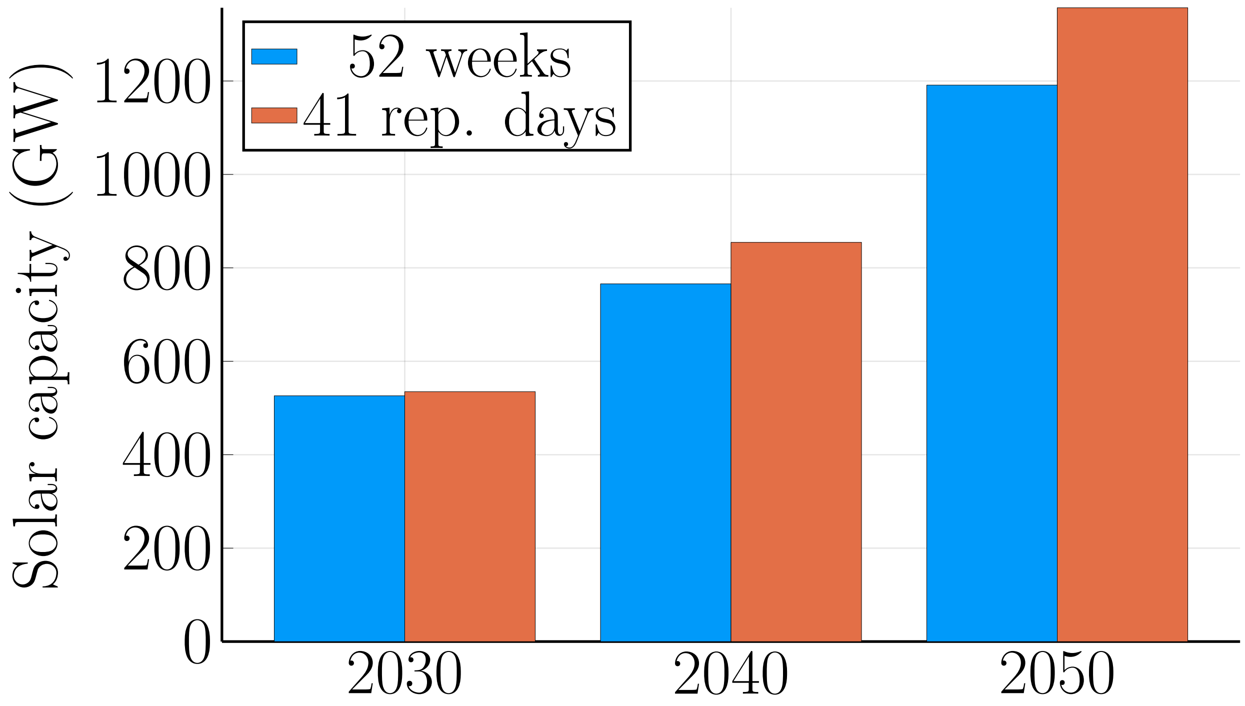
# Performance with discrete investment decisions

*Model solved using 6 computing nodes with 26 cores each for a total of 156 cores. During the first step, we use the interior point regularization function, i.e. R=0*



# Higher temporal resolution reduces modeling errors

We compare against a GenX model using 41 representative days per planning stage.



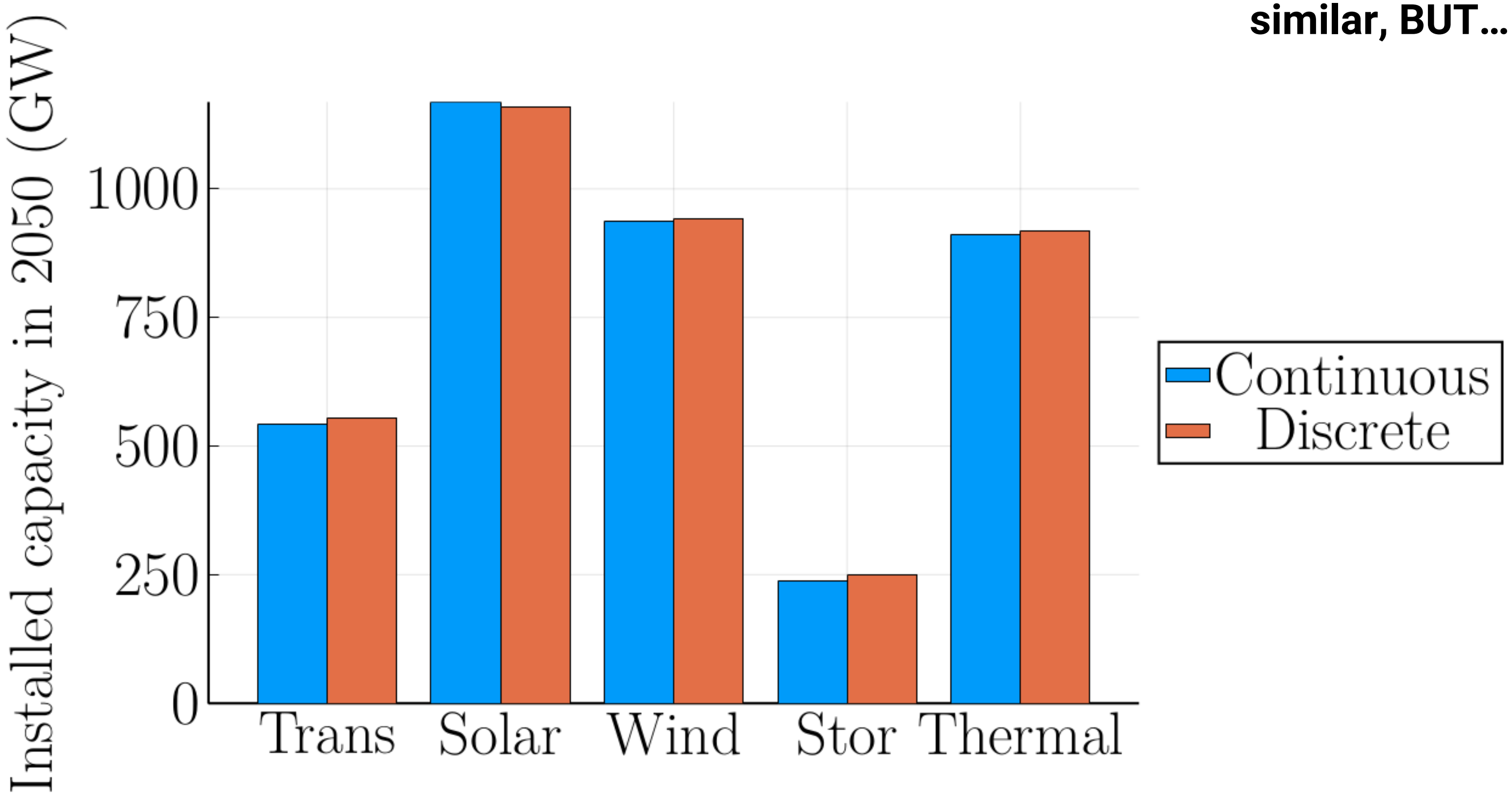

- Using only 41 representative days GenX overestimated the installed solar capacity by 15%.
- Runtime was roughly 6 hours, the same time it took our new Benders decomposition method to solve the same model with full temporal resolution (52 weeks per stage).

*Note that this is a similar temporal resolution to many multistage planning models. For example, the* ***RIO model (Evolved Energy Research):***

- *Best current representation of electricity sector in a multi-sector planning model.*
- *Used in Net-Zero America, Net-Zero Australia, REPEAT Project, Annual Decarbonization Perspectives, many others.*

# What about continuous vs discrete investment decisions?

**Total installed capacity may be very similar, BUT…**

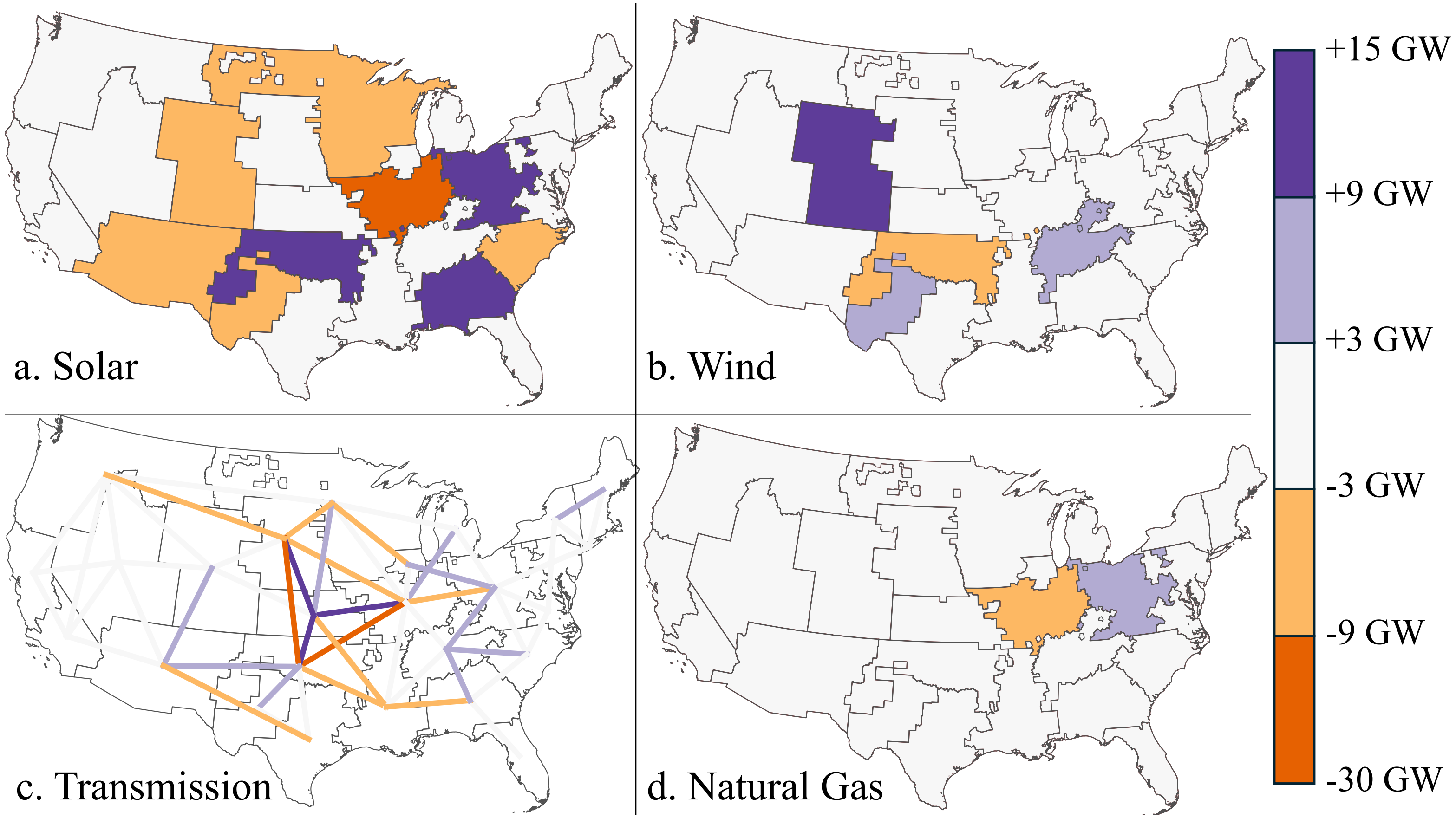



**Note that while installed transmission capacity increases with discrete decisions, cost for transmission expansion actually decreases by ~1%, because the larger installed lines have lower costs.**

# …there are significant regional differences

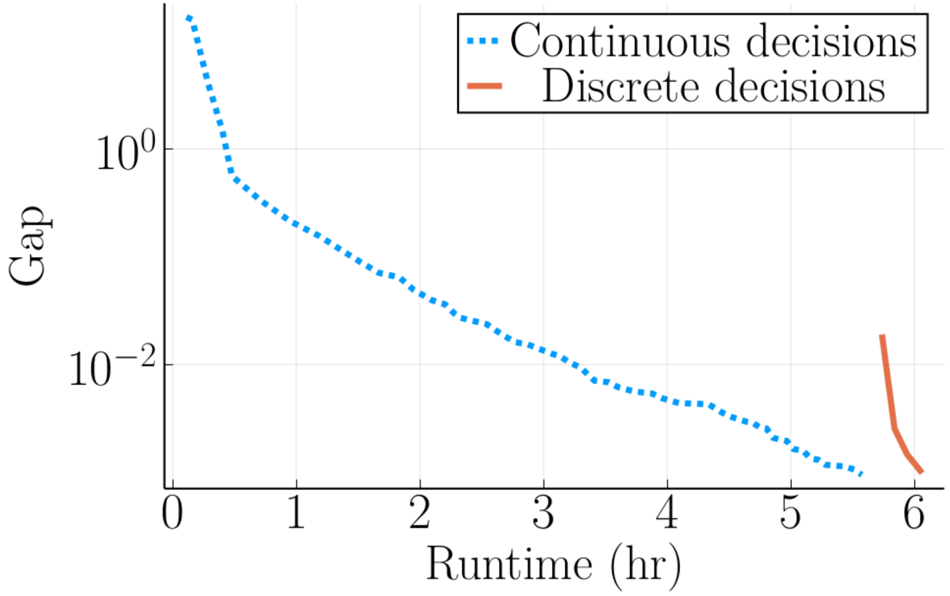
**Figure shows difference in generation and transmission capacity between discrete and continuous cases. Positive values indicates larger installed capacity when discrete decisions are used.**

**The largest differences in transmission capacity corresponds to areas with significant differences in installed renewable energy generation capacity.**



# Strengths

- **Runtime scales linearly** with operational periods (e.g., weeks), enabling planning with multiple weather years or planning stages with 8,736 hours.
- **Discrete investment/retirement decisions** capture economies of unit scale (e.g., transmission lines)
- Accurate modelling of long-duration energy storage & reservoir hydro




*We have solved a three-period energy planning model for a continental United States system with 27 zones and hourly, full-year temporal resolution with 52 weeks for each planning period (e.g., 8,736 hours per planning stage).*

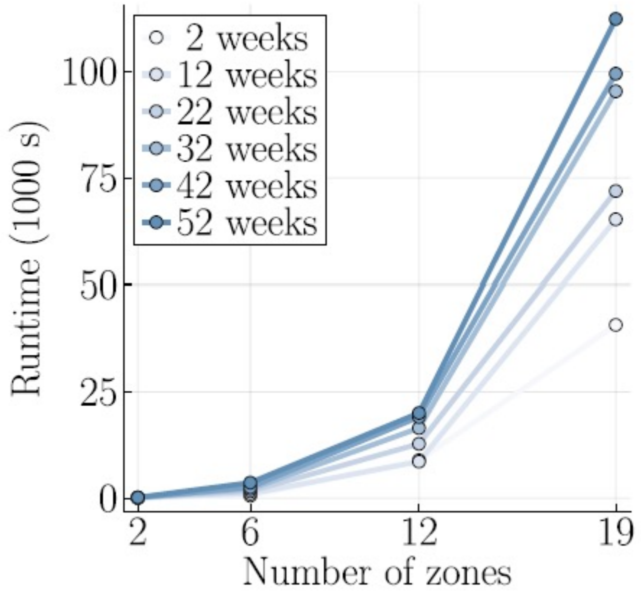
# Limitations

- The model uses linearized power flow (often a simple lossy transport model). Especially when considering DC transmission lines, we should model transmission expansion with DC-OPF equations.

  Ideally, we should link with full fledged AC-OPF operational model to validate results / update transmission constraints. (**Join us!**)

- Runtime still scales quadratically with electricity network size. Future work should focus on incorporating network decomposition within our Benders framework. (Again, **Join us!**)

- Harnessing these advances requires access to distributed computing resources. We need a cloud-based implementation to enable high-performance capacity expansion models for all users.



# Questions and discussion

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